

A Fourth Order Compact Scheme for Transport Equation with Discontinuous Coefficients

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Abstract. This paper reports a compact fourth order accurate cubic spline collocation procedure for numerical solution of transport equation with discontinuous coefficients. The proposed scheme is shown to be stable. A new patch up technique is developed near the interface where the coefficient has jump discontinuity. A numerical experiment is given to confirm the accuracy and efficiency of the proposed scheme.

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1. Introduction

Consider the transport equation in one dimension

$$u_t = c(x)u_x, \quad t > 0, \quad x \in [a, b], \quad (1.1a)$$

$$u(x, 0) = u_0(x), \quad x \in [a, b]. \quad (1.1b)$$

Here c is piecewise smooth having discontinuities across some interfaces within the solution domain. Transport equations with discontinuous coefficients arise in conservation laws. Recently this problem has created a lot of interest among researchers. The primary difficulty in using high order schemes is finding stable scheme at the interface. In recent years, many contributions [1–5] have been made to solve interface problems. Wiegmann

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