

A High Order Compact Scheme for a Thermal Wave Model of Bio-Heat Transfer with an Interface

Suruchi Singh^{1,*}, Swarn Singh² and Zhilin Li³

¹ Department of Mathematics, Aditi Mahavidyalaya, University of Delhi,
New Delhi 110039, India

² Department of Mathematics, Sri Venkateswara College, University of Delhi,
New Delhi 110021, India

³ Department of Mathematics, North Carolina State University, Raleigh,
NC 27695-8205, USA

Received 12 April 2017; Accepted (in revised version) 22 October 2017

Abstract. In this paper, a high order compact difference scheme has been developed for second order wave equations with piecewise discontinuous coefficients. The idea presented here can be used to solve a wide variety of hyperbolic models for non-homogenous inner structures. Thermal wave model of bio-heat transfer in a multilayered skin structure with different thermal and physical properties is investigated subject to constant, linear, exponential and sinusoidal heating. The success of the new procedure is demonstrated by solving test problem as well as by application to the triple layered skin structure.

AMS subject classifications: 65M06

Key words: Compact finite difference scheme, thermal wave bio-heat model, immersed interface method, jump condition, discontinuous coefficients.

1. Introduction

Skin is the largest organ of human body which plays an important role such as defense, sensory and thermoregulation etc. Skin burns are one of the most devastating injuries encountered in human's daily life. These injuries usually result from heat, electricity, radiation, or chemicals. To understand and accurately predict tissue damage following a burn, bio-heat transfer based mathematical model [1–3] of the skin were developed. Investigation of such bio-heat transfer models is always demanding. For bio-heat transfer, Pennes [2] proposed a model based on classic Fourier's Law which is

$$q(x, t) = -k \nabla T(x, t), \quad (1.1)$$

*Corresponding author. Email addresses: ssuruchi2005@yahoo.co.in (S. Singh), ssingh@svc.ac.in (S. Singh), zhilin@ncsu.edu (Z. L. Li)