Journal of Computational and Applied Mathematics 407 (2022) 113975

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A new patch up technique for elliptic partial differential equation with irregularities

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ARTICLE INFO

Article history:
Received 16 October 2020
Received in revised form 12 October 2021

MSC:
primary 65N12
secondary 35K20

Keywords:
Elliptic partial differential equation
Interface
Jump conditions
Cubic spline collocation
Irregularities
Patch up technique

ABSTRACT

This paper presents a new technique based on a collocation method using cubic splines for second order elliptic equation with irregularities in one dimension and two dimensions. The differential equation is first collocated at the two smooth sub domains divided by the interface. We extend the sub domains from the interior of the domain and then the scheme at the interface is developed by patching them up. The scheme obtained gives the second order accurate solution at the interface as well as at the regular points. Second order accuracy for the approximations of the first order and the second order derivative of the solution can also be seen from the experiments performed. Numerical experiments for 2D problems also demonstrate the second order accuracy of the present scheme for the solution u and the derivatives u_x, u_{xx} and the mixed derivative u_{xy}. The approach to derive the interface relations, established in this paper for elliptic interface problems, can be helpful to derive high order accurate numerical methods. Numerical tests exhibit the super convergent properties of the scheme.

1. Introduction

Let Ω be a bounded domain and Γ be an interface. The interface Γ partitions the domain Ω into two non overlapping sub domains Ω^- and Ω^+ so that $\Omega = \Omega^- \cup \Omega^+ \cup \Gamma$. We consider the following second order elliptic partial differential equation of the form:

$$\nabla \cdot (\mu(x)\nabla u(x)) - \nu(x)u(x) = f(x), \quad x \in \Omega \setminus \Gamma, \tag{1.1}$$

subject to boundary conditions defined on the smooth boundary $\partial\Omega$. The functions μ, ν, u and f are assumed to be sufficiently smooth in the two sub domains and may be discontinuous across the interface. The solution and flux jump conditions across the interface are given by

$$[u]_\Gamma = \tau \quad \text{and} \quad [\mu u_n]_\Gamma = \gamma, \tag{1.2}$$

where τ and γ are known and $u_n = \nabla u.n$ is the normal derivative of u and n is the unit normal vector with the direction pointing towards Ω^+ side. These type of equations arise in various mathematical models [1,2] and hence, a wide range of numerical methods have been developed to solve them. Over the past decades, there have been many advances made in the formulation, analysis and application of finite difference and finite element methods for partial differential equations.

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https://doi.org/10.1016/j.cam.2021.113975