



Cubic B-spline method for non-linear sine-Gordon equation

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Abstract

In this article, one dimensional non-linear sine-Gordon equation has been studied. We propose a new method based on collocation of cubic B spline to find the numerical solution of non-linear sine-Gordon equation with Dirichlet boundary conditions. The method involves high order perturbations of the classical cubic spline collocation method at the partition nodes. The existence and uniqueness of the numerical solution has been proved. The method produces optimal fourth order accurate solutions. Stability analysis of the method has been done. The numerical experiments confirm the expected accuracy. We compare the results obtained by our method with those by other techniques for some already existing examples in the literature.

Keywords Sine-Gordon equation · High order · Cubic B spline · Collocation · Stability · Non-linear

Mathematics Subject Classification 65M70

1 Introduction

The sine-Gordon equation is a non-linear hyperbolic partial differential equation that includes sine of sufficiently differentiable functions. This equation appeared first in differential geometry, but later it has been studied in many fields of science because of its exhibition of an important phenomena. The sine-Gordon equation models the structure of solitary waves, that is its solution shows the property of regaining their original shape and velocity after collision. The sine-Gordon equation finds its importance in physics in dislocation theory in crystals, where the term *sinu* appears because of the periodic structure of rows of atoms and in the propagation of fluxons in Josephson junctions. This equation is also used to describe

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