International Journal of Physics and Mathematics

E-ISSN: 2664-8644
P-ISSN: 2664-8636
IJPM 2025; 7(1): 50-56

www.physicsjournal.net
Received: 15-02-2025
Accepted: 14-03-2025

Mainak Mukherjee
Department of Mathematics, Sri Venkateshwara College, University of Delhi South Campus, New Delhi, India

Saureesh Das
Institute of Informatics and Communication, University of Delhi South Campus, New Delhi, India

AK Chaudhary
Department of Physics, Sri Venkateshwara College, University of Delhi South Campus, New Delhi, Delhi, India

LM Saha
Institute of Informatics and Communication, University of Delhi South Campus, New Delhi, Delhi, India

Pankaj Narang
Department of Physics, A.R.S.D. College, University of Delhi South Campus, New Delhi, Delhi, India

MK Das
Institute of Informatics and Communication, University of Delhi South Campus, New Delhi, Delhi, India

Corresponding Author:
Mainak Mukherjee
Department of Mathematics, Sri Venkateshwara College, University of Delhi South Campus, New Delhi, Delhi, India

Nonlinear dynamics and chaos in fractional duffing system

Mainak Mukherjee, Saureesh Das, AK Chaudhary, LM Saha, Pankaj Narang and MK Das

DOI: https://www.doi.org/10.33545/26648636.2025.v7.i1a.107

Abstract
Dynamical complexity of a Duffing oscillator in the presence of both a linear and fractional damping and excited harmonically has been investigated. The potential energy term involve in the system comprises two minima i.e., double well potential. Numerical simulation has been carried out with a view to understand the complex dynamics of such a fractional Duffing system with (*i*) increased values of the fractional order, α and (ii) increased value of the amplitude, F_0, of the external harmonic excitation while retaining the values of other parameters of the system. Time series, phase portraits have been extensively used to demonstrate pictorially the complex dynamics of the system. In addition, Lyapunov exponents have been computed in each of the cases (i) and (ii) to quantitatively asses the involved dynamical complexity.

Keywords: Thermoelectronics, semiconductor electronics, Stanislav Ordin

1. Introduction
A Duffing system has attracted the attention of several research in recent years [1]. Fractional calculus, though introduced almost three hundred years ago, introduces the possibility of differential and integral operators of arbitrary orders. In view of the observed complex frequency dependence of various damping materials, fractional calculus finds applications in many areas of engineering and physics [2, 3]. Fractional calculus has further been introduced earlier to understand the intricacies involved in controlling the dynamics of electronic oscillator such as Chua's circuit etc. [4]. Unlike the ordinary dynamical system requiring at least third order to exhibit chaotic dynamics, fractional order system is found to show chaotic dynamical pattern even when total fractional order is less than three [5, 6]. In the present work, we investigate the dynamical complexity of the Duffing system incorporating both linear and fractional damping terms. Using the various tools of nonlinear dynamics, both regular and chaotic behaviour of the system has been studied using phase space diagrams and Lyapunov characteristic exponent (LCE). The numerical simulation of the fractional Duffing system has been done when (i) fractional order of the system is varied and (ii) the amplitude of the sinusoidal/ harmonic term is varied.
In section 2, we briefly outline the mathematical formulation of the problem. The numerical simulation has been done in section 3. Results obtained are summarized in section 4.

2. Mathematical Formulation of the harmonically excited Fractional Duffing Equation
A Duffing oscillator with a linear damping term and when excited by a harmonic term may be written as:

$$\frac{d^2X}{dt^2} + \alpha\frac{dX}{dt} + \mu X + \rho X^3 = F_0 cos(\omega\, t) \tag{2.1}$$

Where, α is the coefficient of linear damping term, μ and ρ refers to the linear and cubic stiffness term respectively. Further, F_0 corresponds to the amplitude of the harmonic excitation with frequency, ω. The potential energy of the foregoing oscillator is given by