

# Mathematical Analysis of a Prey–Predator Model in Presence of Two Controls



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**Abstract** In this paper, we develop a two-dimensional Lotka–Volterra prey–predator model based on crop and pest. Pontryagin’s maximum principle is deployed to address the control issue of the prey–predator system, with the interaction of the coefficients of the Lotka–Volterra equations as the main input. The optimal control techniques are utilized to enhance agricultural yields while decreasing the predator population, which harms the crops. Control variables are utilized in two ways: one is applied to the predator, which kills the predator, and the other is applied to the crops, which support plant development and soil fertility. The rate of sprayed pesticides is the first control variable and the second control variable is the regularity of usage of organic biomass fertilizers. Furthermore, we establish the existence of an optimal control, the required work out the criteria for optimality, and then carry out numerical computations.

**Keywords** Pest · Crop · Pest control · Prey–predator model

## 1 Introduction

Punjab, with a massive landmass of 1.5% of the total geographical area of India, and much of this being under agricultural use, has always been a crucial contributor to Indian economy. Punjab has been able to provide a significant boost to the country’s food security. As a result, the state plays a significant role in the economic development of India. During the last few years, the role of agriculture and allied sectors has decreased in Gross State Value Added (GSVA) from 30.81% (2011–12) to 27.22% in 2014–15, in which the share of crops declined from 19.73% (2011–12) to 16.76% (2014–15) [20]. However, due to systemic difficulties, agriculture during the Green Revolution has also become one of the primary causes of environmental danger in Punjab. Profitability increased several times as a result of the implementation of this

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