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Aequationes Mathematicae

Transitivity and sensitivity of iterated function systems via Furstenberg families

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Abstract. In this paper, we study variants of transitivity and sensitivity via Furstenberg families for iterated function systems (IFSs). Using the concept of skew product transformation of an IFS, we obtain results related to characterizations of the concepts studied. Also results regarding the inverse, conjugacy and product of IFSs are studied. Wherever necessary examples and counterexamples are provided related to the results obtained.

Mathematics Subject Classification. 37B05, 37B20, 37D45, 54H20.

Keywords. Iterated function system, Transitivity, Mixing, Sensitivity, Product map.

1. Introduction

In the theory of dynamical systems, chaos is an important phenomenon. Among the several definitions of chaos, Devaney's definition which consists of three conditions namely transitivity, dense periodic points and sensitivity [8] is widely accepted. In the study of chaos, the concepts of transitivity and sensitivity are key ingredients. They are of great importance in the qualitative study of dynamical systems. The concept of transitivity can be traced back to Birkhoff [4]. It is also regarded as an irreducibility condition because one cannot decompose a transitive dynamical system into two disjoint sets with non-empty interiors so that they do not interact under the transformation. It is the basis for the study of several decomposition theorems. Sensitivity, also known as butterfly effect, characterizes the unpredictability of chaotic phenomena. If a dynamical system is sensitive, then a small change in the initial conditions leads to a significant change in the dynamics of the system. These days, the sensitivity theory of dynamical systems is being widely studied by many researchers [12,23]. While studying the transitivity or sensitivity of dynamical systems one may encounter a natural question: "Do they happen