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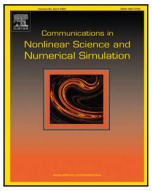

Research paper

Sensitivity and unpredictability in semiflows on topological spaces

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ABSTRACT

We introduce and study notions of topological sensitivity and topologically unpredictable point for semiflows on topological spaces. Both these notions are related with their original versions. With the help of topological transitivity, we provide some sufficient conditions for a semiflow to be topologically sensitive. The notion of topologically unpredictable point is studied on the arbitrary product of semiflows and on hyperspatial semiflows. We also see when every point in the trajectory of a topologically unpredictable point is topologically unpredictable. We provide a result associating Poincaré chaos with topological sensitivity. The relation of Poincaré chaos with some other types of chaos related with unpredictability is also studied. Some examples and counterexamples are also provided to justify our results obtained.

1. Introduction

Dynamical system is one of the important and widely applicable branches of mathematics. Its theory mainly focuses on the long-term behaviour of systems that are evolving over time according to fixed set of rules. Applying techniques of this theory, long-term behaviour of many daily life occurrences such as the motion of celestial bodies in our solar system, population growth and decline, weather forecast, and many more can be approximated with a high level of accuracy. There was a motivation to generalize the comprehensive context of topological semigroups so as the classical case of dynamical system arises as a particular instance. This idea led to the study of semiflows. In its study, chaos is one of the foremost aspects which we come across. Mathematically, chaos was initially studied in 1975 by Li and Yorke [1]. Thereafter, many researchers have introduced and studied various definitions of chaos [2–4]. Devaney chaos [5] is the most famous among them. It consists of three things, namely, topological transitivity, denseness of periodic points and sensitivity. Transitivity is considered as an irreducibility condition. Sensitivity, famously known as the butterfly effect, is the key ingredient of chaos. It is essential to study sensitivity as it characterizes the unpredictability of a chaotic phenomenon.

The notion of sensitivity was initially studied by Auslander and Yorke [3] in 1980. Once it is known to us that a system is sensitive, it motivates us to study the strength of sensitivity. For this, many stronger notions of sensitivity have been introduced and studied by several researchers [6–8]. Li and Zhou [9] researched how these notions carry on the product and vice versa. Many of these works were studied by considering phase spaces as metric spaces. It is quite noteworthy that to extend many concepts related to sensitivity to non-metrizable phase spaces, Good and Macías [10] suggested two feasible ways. One is through uniformities and the second is with the help of open covers. In 2018, using open covers, Fedeli [11] considered sensitivity in discrete dynamical systems on topological spaces. The author related the definition with the metric version and also provided some sufficient conditions for a system to be topologically sensitive.

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