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Research Article

Radii of convexity associated with various subclasses of analytic functions for functions related to Kaplan classes

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Abstract

A normalized analytic function f defined on the open unit disc $\mathbb{D}$ is called Ma-Minda convex if $1+zf''(z)/f'(z)$ is subordinate to the function φ. For $0 \leqslant \alpha \leqslant \beta$, the Kaplan class $\mathcal{K}(\alpha, \beta)$ of type α and β consists of normalized analytic functions of the form $p^{\alpha}g$ defined on $\mathbb{D}$ where p with $p(0) = 1$ is an analytic function taking values in the right half-plane and g is an analytic function with $g(0) = 1$ satisfying $\mathrm{Re}(zg'(z)/g(z)) > (\alpha-\beta)/2$. For functions f with $f' \in \mathcal{K}(\alpha, \beta)$, we obtain the radius of Ma-Minda convexity for various choices of φ. The radius of lemniscate convexity, lune convexity, nephroid convexity, exponential convexity and several other radius estimates are examined. The results obtained are sharp.

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1. Introduction

Let $\mathcal{A}$ denote the class of all analytic functions f defined on the open unit disc $\mathbb{D}$ and normalized by the conditions $f(0) = 0$, $f'(0) = 1$. Let $\mathcal{A}_0$ be the class of functions g such that $zg \in \mathcal{A}$, this class consists of analytic functions with $g(0) = 1$. The set of univalent (one-to-one) functions in the class $\mathcal{A}$ is symbolized as $\mathcal{S}$. For $0 \leqslant \alpha < 1$, let $\mathcal{ST}(\alpha)$ and $\mathcal{CV}(\alpha)$ denote the subclasses of the class $\mathcal{A}$ consisting of starlike and convex functions respectively such that $\mathrm{Re}(zf'(z)/f(z)) > \alpha$ and $\mathrm{Re}(1 + (zf''(z)/f(z))) > \alpha$. For $\alpha = 0$, these classes reduces to the class of Starlike and convex functions denoted by $\mathcal{ST}$ and $\mathcal{CV}$, respectively. Many classes in geometric function theory can be unified using the so called Kaplan classes. Define

$$\mathcal{H} := \{f \in \mathcal{A}_0 : \text{for some} \quad \delta \in \mathbb{R}, \mathrm{Re}\, e^{i\delta} f(z) > 0, z \in \mathbb{D}\}.$$

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