Monatshefte für Mathematik
https://doi.org/10.1007/s00605-024-02013-6

Lappan's five-point theorem for φ-normal harmonic mappings

Nisha Bohra[1] · **Gopal Datt**[2] · **Ritesh Pal**[2]

Received: 26 June 2024 / Accepted: 11 August 2024

Abstract
A harmonic mapping $f = h + \overline{g}$ in $\mathbb{D}$ is φ-normal if $f^{\#}(z) = \mathcal{O}(|\varphi(z)|)$, as $|z| \to 1^{-}$, where $f^{\#}(z) = (|h'(z)| + |g'(z)|)/(1 + |f(z)|^2)$. In this paper, we establish several sufficient conditions for harmonic mappings to be φ-normal. We also extend the five-point theorem of Lappan for φ-normal harmonic mappings.

Keywords Normal functions · Normal harmonic mappings · φ-normal functions · Spherical derivative

Mathematics Subject Classification 30D45 · 31A05

1 Introduction and main results

The notion of *normal functions* was introduced by Yosida in [21], and later studied by Noshiro, although they did not use the term normal function. For the class of normal functions both of them used the term "**class (A)**". The name **normal function** was given by Lehto and Virtanen in their pioneering paper [13], wherein they showed that the normal functions are closely related to the problems of boundary behaviour of meromorphic functions. Let us recall the definition of normal function. A meromorphic function f on the unit disc $\mathbb{D} \subset \mathbb{C}$ is normal if the family $\{f \circ \varphi : \varphi \in \mathsf{Aut}(\mathbb{D})\}$ is normal in the sense of Montel in $\mathbb{D}$, where $\mathsf{Aut}(\mathbb{D})$ is the group of conformal automorphisms of the unit disc. Noshiro established a characterization of normal

Communicated by Adrian Constantin.

✉ Gopal Datt
ggopal.datt@gmail.com; gopal.du@bbau.ac.in

Nisha Bohra
nishabohra@svc.ac.in

Ritesh Pal
rriteshpal@gmail.com

[1] Department of Mathematics, Sri Venkateswara College, University of Delhi, Delhi, India

[2] Department of Mathematics, Babasaheb Bhimrao Ambedkar University, Lucknow, India

Published online: 17 October 2024