DOI: 1061294/jiaps2023.2734 ISSN 0974 - 9373

Vol. 27 No. 3 (2023) **J. Int. Acad. Phys. Sci.** pp. 237-247

Numerical Solution of Natural Convection Nanofluid Flow over a Non-Isothermal Vertical Plate

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(Received April 27, 2023, Accepted August 30, 2023)

Abstract: The natural convection nanofluid flow over a non-isothermal vertical plate is studied numerically. The effects of Brownian motion and thermophoresis parameters are incorporated into models used for nanofluids. The non-linear partial differential equations and boundary conditions are transformed into a set of nonlinear ordinally differential equations using the similarity transformations. The resulting system of equations are solved numerically by using shooting techniques. This solution depends on the Prandtl number (Pr), Buoyancy ratio (Nr), Brownian- motion (Nb), thermophoresis parameter (Nt), Lewis number (Le), and power-law exponent (λ). For different values of λ, Le and Pr, the influence of reduced Nusselt number with Nr, Nb and Nt is represented by correlation formulas and compared with previously published results and found to be in better agreement.

Keywords: Brownian motion, Nanofluid, Natural Convection, Non-isothermal, Power law exponent, Thermophoresis.

1. Introduction

The keyword "nanofluid" was used to explain a fluid with nanometer-sized particles suspended in a principal heat transmission fluid. Nanofluid can be employed in various applications, including electronics, communications, medicine, apparatus for the optical, lasers, X- rays with high energy, scientific evaluation, processing of materials, technologies of computation, and synthesis of materials, which has sparked a lot of interest. The term nanofluid was first used by Choi[1]. The phenomenon found by Masuda *et al.*[2] is that nanofluids increase thermal conductivity. This phenomenon indicates the potential for taking advantage of nanofluids in advanced nuclear systems, as given in Buongiorno and Hu[3]. Wang and Mujumdar[4] reviewed the heat transmission characteristics of nanofluids. Later, Das and Choi[5] surveyed the general topic of heat transmission in nanofluids.

Natural convection flows have attracted much attention due to their numerous technical and industrial uses in the heat transfer process. Convection cells, which develop as air rises over sun-warmed land or water, are a common component of all-weather systems in nature. Convection can also be seen in the rising plume of hot air produced by a fire, oceanic currents, and the creation of sea winds (where Coriolis forces also modify upward convection). Convection is typically depicted in engineering applications as creating microstructures during the cooling of molten metals and fluid flows around shrouded heat-dissipation fins and solar ponds. Free air cooling without fans is a relatively widespread industrial application of natural convection, which can occur on small sizes (computer chips) to large-scale process equipment. Natural convection transmits heat in various physical and technical applications, including geothermal systems, heat exchangers, chemical catalytic reactors, fibre and granular insulation, packed beds, petroleum reservoirs, and nuclear waste storage. Furthermore, natural convection of heat and mass transmission occurs in many regions. It can be found, for example, in the fields of chemical processing equipment design, for production and dispersion, temperature and moisture distributions over agricultural fields and fruit tree groves. Crop damage due to freezing and environmental contamination.

Natural convection is beneficial fluid passing through a vertical plate in a familiar environment, a classical problem. It was initially examined theoretically by E. Pohlhausen for contributions to Schmidt and Beckmann[6] experimental work. Subsequently, Khair and Bejan[7] expanded the work of heat and mass transmission. Using a combined similarity numerical approach, Khan and Aziz[8] investigated a problem involving a nanofluid that flows naturally over a constant heat flux on a vertical plate. After that, Aziz and Khan[9] again used an equivalent approach to review the natural convective physical phenomenon flow of a nanofluid over a vertical plate under a convective condition. Kuznetsov and Nield[10] study the impact of nanoparticles on natural convective boundary layer flow across a vertical plate using an analytical model that took into account Brownian motion and thermophoresis. Then, they modified the problem so that the volume fraction of nanoparticles at, instead of being actively regulated, the boundary is seen to be passively controlled[11].

Natural convection via a non-isothermal vertical plate during a porous medium flooded with a nanofluid was examined by Gorla and Chamkha[12]. Bagai and Nishad[13] explored the impact of variable temperature profile and fluid index on the velocity and temperature profile for a plate horizontally implanted within the confines of a porous medium immersed with fluids that aren't Newtonian when there is a source of internal heat. Khan *et al.*[14] study about flow of a viscous nanofluid flowing stretching surface. Non-linear thermal radiation, heat source/sink, and dense, and incompressible factors are considered in the analysis. Utilizing Darcy's law, Ibrahim and Khan[15] investigated the mixed convection of nanofluids in a porous media across a transparent stretched sheet. Pal and Mandal analyzed the MHD convection flow of nanofluid over such a stretching sheet toward a static pressure[16]. Checking magneto-hydrodynamics boundary layer Au-water nanofluid flow across a shrinking porous surface using the thermal radiation effect was given by Rashid *et al.*[17].

Motivated from the above discussion, we extend the study of Kuznetsov and Nield[11]. We are applying Buongiorno[18] nanofluid model to obtain the similarity solutions. These results depend upon the following parameters: Prandtl number (Pr), power law exponent (λ), Buoyancy-ratio (Nr), Brownian motion (Nb), Lewis number (Le), and thermophoresis (Nt).

2. Mathematical Formulation

We consider the steady two-dimensional coordinate system (x,y) with the x-axis measured along the plate in the upward direction and the y-axis masured in the normal direction to the plate. The wall temperature of the vertical plate is the power function of distance from the origin, and the flux of nano-particle fraction is zero. For away from the plate, the ambient values of temperature and nanoparticle fraction are denoted by φ_∞ and T_∞, respectively. Figure 1 displays the physical model and coordinate systems.

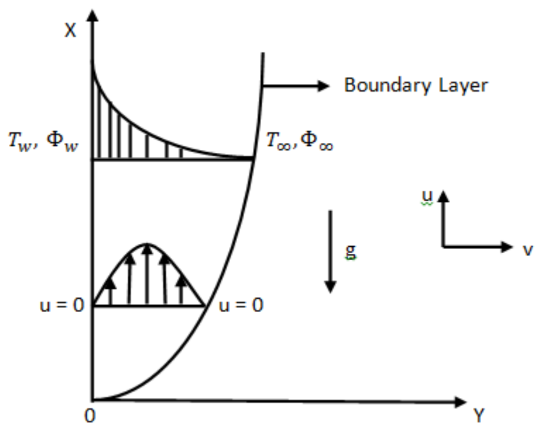

Figure 1: Physical model and coordinates system

Applying Oberbeck Boussinesq approximation[19], the governing equations for total mass, momentum, thermal energy, and nanoparticles conservation are as follows[10]

(1) $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(2) $$\rho_f\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\left[(1-\phi_\infty)(\mathrm{T}-\mathrm{T}_\infty)\beta\rho_{f\infty}-(\phi-\phi_\infty)\left(\rho_p-\rho_{f\infty}\right)\right]g$$

(3) $$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial \phi}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right]$$

(4) $$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D_B\frac{\partial^2 \phi}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}$$

where

$$\alpha=\frac{k}{(\rho c)_f},\ \tau=\frac{(\rho c)_p}{(\rho c)_f}.$$

Here u and v are the velocity components of x and y direction respectively, ρ_f is the density of the base fluid, μ is the viscosity of the base fluid, β is the volumetric thermal diffusivity of the base fluid, ρ_p is the density of the nanoparticles, g is the acceleration due to gravity, α is the thermal diffusivity of the base fluid, k is the thermal conductivity of the nanofluid, τ is the ratio the nanoparticles heat capacity and the base fluid heat capacity, φ is the local solid volume fraction of the nanofluid, T is the local temperature, D_B is the Brownian diffusion coefficient, and D_T is the thermophoresis diffusion coefficient. The subscript ∞ represents the values at $y\to\infty$ where the fluid is quiescent. The equations (1)-(4) must be solved by using the following boundary conditions

(5) $$u=\mathrm{v}=0,\mathrm{T}=\mathrm{T}_w=\mathrm{T}_\infty+Ax^\lambda,\mathrm{D}_B\frac{\partial\phi}{\partial y}=-\left(\frac{D_T}{T_\infty}\right)\frac{\partial T}{\partial y},\mathrm{x}\geq 0,\text{ at }\mathrm{y}=0$$

(6) $$u=\mathrm{v}=0,\mathrm{T}=\mathrm{T}_\infty,\phi=\phi_\infty,\ \text{ as }\mathrm{y}\to\infty$$

where $A>0,$ is constant, and the wall temperature of the vertical plate is a power function of distance from the origin.

Now introducing stream function, which defined as

$$u=\frac{\partial\psi}{\partial y},\mathrm{v}=-\frac{\partial\psi}{\partial x}$$

Substituting u and v into equations (1) - (4) we have

(7) $$\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)-\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial x\,\partial y}\right)+v\frac{\partial^3\psi}{\partial y^3}=\frac{(\phi-\phi_\infty)\left(\rho_p-\rho_{f\infty}\right)g}{\rho_{f\infty}}-(1-\phi_\infty)\beta(T-T_\infty)g$$

(8) $$\left(\frac{\partial T}{\partial x}\right)\left(\frac{\partial\psi}{\partial y}\right)-\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial\psi}{\partial x}\right)=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial\phi}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right]$$

(9) $$\left(\frac{\partial\phi}{\partial x}\right)\left(\frac{\partial\psi}{\partial y}\right)-\left(\frac{\partial\phi}{\partial y}\right)\left(\frac{\partial\psi}{\partial x}\right)=D_B\frac{\partial^2\phi}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2},$$

here $\upsilon=\frac{\mu}{\rho_{f\infty}}$, further, define dimensionless similarity variables

$$s(\eta)=\frac{\psi}{\alpha Ra_x^{1/4}},\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\mathrm{f}(\eta)=\frac{\phi-\phi_\infty}{\phi_w-\phi_\infty},\eta=\frac{y}{x}Ra_x^{1/4}$$

where Ra_x is called the local Rayleigh number and defined as

$$Ra_x=\frac{(1-\phi_\infty)\beta g(T_w-T_\infty)x^3}{v\alpha}$$

Substituting the values of s,θ and f in equations (7)- (8), we get

(10) $$s'''+\frac{1}{\mathrm{Pr}}\left[\left(\frac{\lambda+3}{4}\right)ss''-\left(\frac{\lambda+1}{2}\right)s'^2\right]-Nrf+\theta=0$$

(11) $$\theta''+\left(\frac{\lambda+3}{4}\right)s\theta'+Nb\theta'f'+Nt\theta'^2-\lambda s'\theta=0$$

(12) $$\mathrm{f}''+\left(\frac{\lambda+3}{4}\right)\mathrm{Lesf}'+\left(\frac{Nt}{Nb}\right)\theta''=0$$

where prime ($'$) represents the derivative of the function concerning η. The Pandtle number, buoyancy ratio, Brownian motion, thermophoresis parameters and Lewis number are defined as

$$\mathrm{Pr}=\frac{v}{\alpha};\quad Nr=\frac{(\rho_p-\rho_{f\infty})(\phi_w-\phi_\infty)}{(1-\phi_\infty)\beta\rho_{f\infty}}(T_w-T_\infty);$$
$$Nb=\frac{\tau D_B(\phi_w-\phi_\infty)}{\alpha};\quad Nt=\frac{\tau D_T(T_w-T_\infty)}{\alpha T_\infty};\quad Le=\frac{\alpha}{D_B}$$

Further, the boundary conditions (5)-(6) are converted as

(13) $$s(0)=0,s'(0)=0,\theta(0)=1,f'(0)=-\left(\frac{Nt}{Nb}\right)\theta'(0)\quad(\eta=0)$$

(14) $$s'(\infty)\to 0,\theta(\infty)\to 0,\mathrm{f}'(\infty)\to 0\quad(\eta\to\infty)$$

If λ = 0, then equations (10)-(12) are same as Kuznetsov and Nield[11]. Furthermore, if λ, Nr, Nb and Nt are equal to zero, these equations contain only two independent variables, s and θ. In this case above problem must be converted into Pohlhausen- Kuiken - Bejan classical problem.

In this study, let us define a physical quantity for practical interest in the form of Nusselt number as

(15) $$Nu=\frac{q_w''}{(T_w-T_\infty)}\left(\frac{x}{k}\right),$$

where q_w'' is called the heat flux of the wall. Further, by using equation (15), we defined another number known as the reduced Nusselt number

(16) $$Nur=\frac{Nu}{Ra_x^{1/4}}=-\theta'(0).$$

3. Results and Discussion

The set of nonlinear ordinary differentials equations (10)-(12) subject to boundary conditions (13) and (14) have been solved numerically by using the shooting method. We must compare our numerical results obtained in this paper with previously published research papers available in the literature. In Table 1, we present the numerical values of the reduced Nusselt number along with the results given by Kuznetsov and Nield[11] for the case of vertical plate, which are found in perfect agreement.

Table 1. Compare the present reduced Nusselt number with previously published results for various Pr when Le = 10, Nr = Nb = Nt = 10^{-5} and $\lambda = 0$.

Pr	1	10	100	1000
Nur (Kuznetsov and Nield [11])	0.401	0.463	0.481	0.484
Nur (Present)	0.4010	0.4633	0.4811	0.4836

After comparing the results, we plot all the dependent similarity variables for a typical case, choosing Pr = 10, Le = 5, Nr = 0.1, Nb = 0.3, Nt = 0.5 and $\lambda = 0$, which are shown in Figure 2. This figure shows that if the fluid is regular, then the boundary layer profiles for the temperature function are the same as the stream function. Further, the momentum boundary layer thickness is greater than the thermal boundary layer thickness when Pr > 1. Subsequently, the thermal boundary layer thickness is more than the thickness of the boundary layer for the mass fraction function when Le > 1. In figures 3-6, we plot the dimensionless velocity, temperature, nanoparticle volume fraction and reduced Nusselt number for various values of power-law exponent (λ) when Pr = 10, Le = 5, Nr = 0.1, Nb = 0.3, Nt = 0.5 and understand the effect of λ.

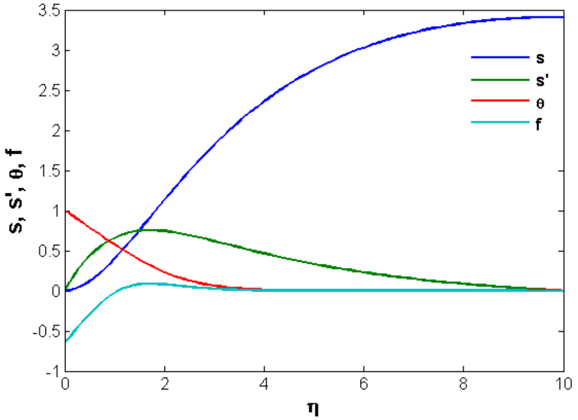

Figure 2: Plots similarity variables for fixed values of Pr = 10, Le = 5, Nr = 0.1, Nb = 0.3, Nt = 0.5 and $\lambda = 0$.

The effect of power-law exponent (λ) on the velocity profile is depicted in Figure 3. It shows that the nanofluid velocity initially increases up to some maximum value and then decreases and approaches zero as the value of $\eta \rightarrow \infty$ (i.e. as the distance from the plate increases). It reaches the boundary conditions at infinity asymptotically.

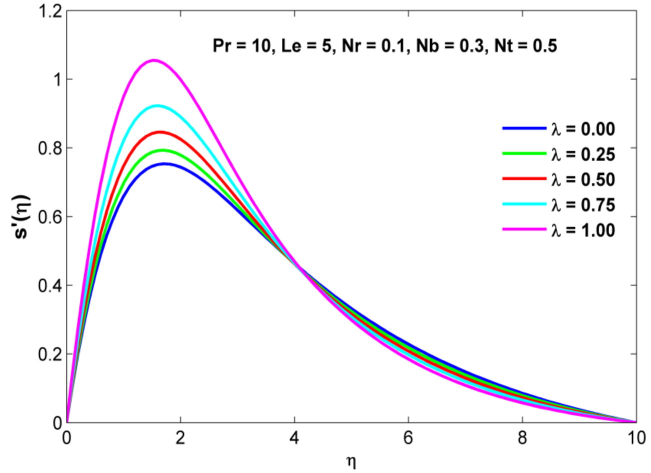

Figure 3: Velocity profiles for various values of the power-law exponent λ.

The temperature profile is shown in Figure 4. It is observed that the temperature increases with the increasing power-law exponent. The power-law exponent λ is directly proportional to the temperature difference.

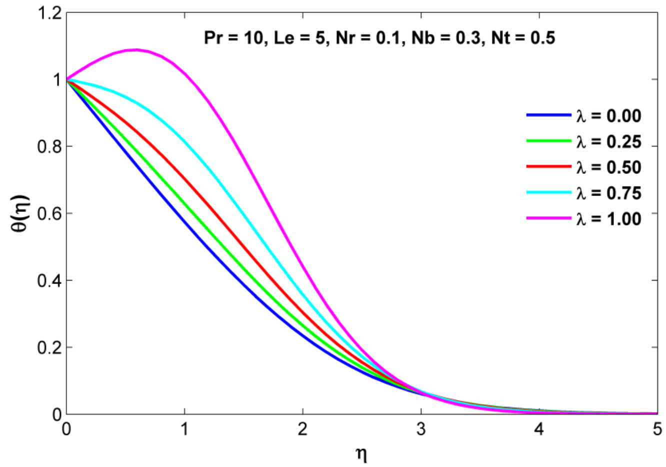

Figure 4: Temperature profiles for various values of the power-law exponent λ.

In Figures 5-6, we can see that the nanoparticle volume fraction and reduced Nusselt number profiles change their nature alternately within the boundary layer when increasing the power-law exponent.

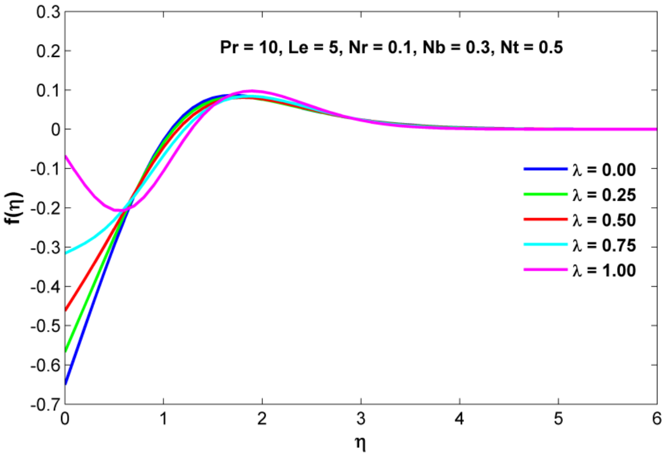

Figure 5: Nanoparticle volume fraction profiles for various values of the power-law exponent λ.

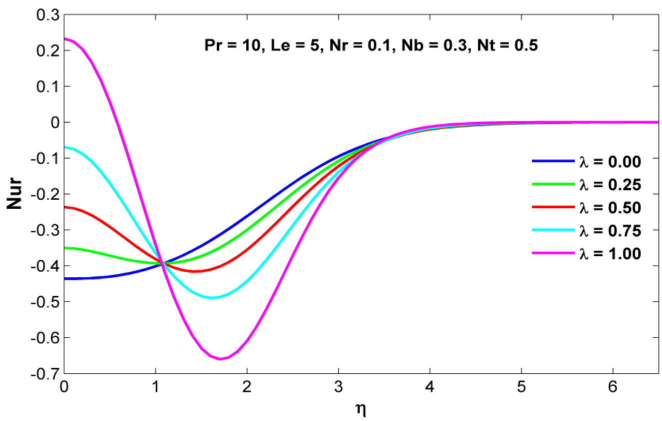

Figure 6: Reduced Nusselt number for various values of the power-law exponent λ.

It is more beneficial to provide the information in tabular form and develop correlations similar to those presented in[9] and[11]. We consider the linear regression model as follows

(17) $$Nur_{est} = Nur + C_r N_r + C_b N_b + C_t N_t$$

where C_r, C_b and C_t are the coefficients of the Buoyancy ratio, Brownian motion and thermophoresis parameter, respectively. The linear regression estimations, which are defined in equation (17), are determined for 125 sets of values of N_r, N_b and N_t in the range [0.1, 0.5] and various matters of power-law exponent λ. Further, we represent the linear regression coefficients, reduced Nusselt number and maximum relative error for different values λ in Tables 2 and 3.

Table 2. Linear regression coefficients and Nur for various values of Pr and λ when Le = 10.

λ	Pr	Nur	C_r	C_b	C_t	ε
0.25	1	0.3270	- 0.0011	0.0014	- 0.0931	0.0003
	10	0.3894	- 0.0002	0.0003	- 0.0979	0.0008
	100	0.4085	- 0.0003	0.0004	- 0.0984	0.0002
	1000	0.3954	- 0.0125	0.0207	- 0.0615	0.0323
0.50	1	0.2403	- 0.0016	0.0019	- 0.1299	0.0004
	10	0.2968	- 0.0009	0.0012	- 0.1416	0.0003
	100	0.3156	- 0.0010	0.0013	- 0.1444	0.0004
	1000	0.3142	- 0.0089	0.0012	- 0.1336	0.0134
0.75	1	0.1366	- 0.0011	0.0013	- 0.2014	0.0008
	10	0.1773	- 0.0009	0.0011	- 0.2382	0.0012
	100	0.1921	- 0.0009	0.0011	- 0.2501	0.0014
	1000	0.1799	- 0.0109	0.0003	- 0.2224	0.0157

Table 3. Linear regression coefficients and Nur for various values of Le and λ when Pr = 10.

λ	Le	Nur	C_r	C_b	C_t	ε
0.25	5	0.3889	- 0.0020	0.0010	- 0.0769	0.0029
	10	0.3894	- 0.0002	0.0003	- 0.0979	0.0008
0.50	5	0.2969	- 0.0018	0.0023	- 0.1219	0.0005
	10	0.2968	- 0.0009	0.0012	- 0.1416	0.0003
0.75	5	0.1769	- 0.0017	0.0021	- 0.2133	0.0012
	10	0.1773	- 0.0009	0.0011	- 0.2382	0.0012

These tables show that the reduced Nusselt number decreases as the parameter N_r and N_t increase (C_r, C_t are negative) but increases with an increase in N_b (C_b is positive) for various values Prandtl, Lewis numbers and power-law exponent. The maximum relative error in the last column of Tables 2 and 3 clearly shows that the linear regression models of equation (17) are accurate representations of numerical data.

4. Conclusion

The present work investigates the natural convection nanofluid flow over a non-isothermal vertical plate numerically. The equations governing the flow are expressed in a couple of first-order differential equations using the similarity analysis. This system of equations is then solved numerically by the shooting method. The effect of power-law exponent λ on the dimensionless velocity, temperature, nanoparticle volume fraction and Nusselt number has been analyzed through graphs. These graphs show that as power-law exponent λ increases, the velocity and temperature profile increase, whereas the volume fraction profile changes its nature alternately within the boundary layer. The reduced Nusselt number decreases with increasing power-law exponent, Bounyancy ratio, and thermophoresis, whereas it increases with increasing Brownian motion.

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