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Effect of Power Law Exponent in the Cheng-Minkowycz Natural Convection Nanofluid Flow along a Vertical Plate Embedded in Porous Media

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Abstract: The present study considers the natural convection flow in the Cheng-Minkowycz problem and a vertical plate in a porous medium. Brownian motion and thermophoresis effects are accounted for in the nanofluid model. The Darcy model is applied for the porous medium. A common similarity transformation converts the partial differential system that governs the problem into an ordinary system. The numerical solutions to the ordinary system produce the desired results. These solutions depend on the power-law exponent (λ), buoyancy-ratio (Nr), Lewis number (Le), thermophoresis parameter (Nt) and Brownian motion parameter (Nb). The dependency of the Nusselt number on these five parameters is investigated numerically and graphically. Natural convective heat transfer in porous media finds applications in various industries including solar power, geothermal systems, electronic equipment cooling, nuclear reactors, lubricants, groundwater control, waste disposal, etc..

Keywords: Cheng-Minkowycz problem, Nanofluid, Similarity solution, Natural convection, power law exponent, Porous medium.

1. Introduction

Nanofluid refers to a new class of heat transfer fluid introduced by Choi[1] at Argonne National Laboratory (ANL). A nanofluid consists of liquid suspensions of 1-100 nm ultra-fine solid particles dispersed in a base fluid. The main characteristic feature of nanofluids is thermal conductivity enhancement. This property of nanofluids plays a remarkable role in convective heat transfer processes. The literature on thermal conductivity enhancement of nanofluids was reviewed by Masuda *et al.*[2], Das *et al.*[3], and Chon *et al.*[4], among several others. Geothermal energy recovery, crude oil extraction, groundwater pollution, thermal energy storage, and flow through filtering medium are just a few technical applications for porous media heat transfer challenges.

Buongiorno[5] offered a comprehensive review of convective transport in nanofluids. He pointed out that the suspended nanoparticles' dispersion, turbulence, and particle rotation effects are too small to explain nanofluids convective heat transfer enhancement. Then he developed an exceptional analytical model for the convective transport phenomena in nanofluid by considering seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusionphoresis, Magnus effect, fluid drainage, and gravity setting. He concluded that only Brownian diffusion and thermophoresis are essential mechanisms in nanofluids out of these seven mechanisms. He developed conservation equations for mass, momentum, and heat transport in nanofluids based on these two effects.

The problem of natural convection in a porous medium past a vertical plat is a classical problem first studied by Cheng and Minkowycz[6]. The problem is presented as a paradigmatic configuration and solution in the book by Bejan[7]. The extension to the case of heat and mass transfer was made by Bejan and Khair[8]. Further work on this topic is surveyed in Sections 5.1 and 9.2.1 in Nield and Bejan[9]. The studies by Khaled and Chamkha[10], Chamkha and Quadri[11], and Chamkha and Pop[12] are of special significance. Na and Pop[13] developed a numerical solution for the Cheng-Minkowycz model originating in free convection flow in porous material using the approach of perturbation series in combination with Shanks transformation. Later, Nield and Kuznetsov[14] analyzed the Cheng-Minkowycz problem for natural convective boundary layer flow in a porous medium saturated with nanofluid. Then, they extend this problem to double-diffusive nanofluid convection in a porous medium[15]. Subsequently, Kuznetsov and Nield[16] revised the problem[14] so that the nano-particle volume fraction on the boundary is considered passively rather than actively controlled. By adopting the Buongiorno model[5], Heady *et al.*[17] extend the research of Nield and Kuznetsov[14] to the case when a nanofluid is a non-Newtonian.

A numerical study of natural convection about a vertical cone embedded in a non-Darcian nanofluid containing gyrotactic microorganism-saturated porous medium was studied by Hady *et al.*[18]. Computational investigation of Stefan blowing and multiple slip effects on boundary-driven bioconvection nanofluid flow with microorganisms were studied by Jashim Uddin *et al.*[19]. Hady *et al.*[20] present a numerical study of the problem of unsteady Thermo bioconvection boundary layer flow of a nanofluid containing gyrotactic microorganisms along a stretching sheet under the influence of magnetic field and viscous dissipation. The influence of Brownian motion and thermophoresis on a nonlinearly permeable stretching sheet in a nanofluid was studied by Falana *et al.*[21]. Sheikh and Hasan[22] were to study the mixed convective flow of micropolar fluids in an inclined porous plate with a viscous dissipation effect. Unsteady magneto hydrodynamics mixed convection flow of an electrical conduction nanofluid in a stagnation region of a rotating sphere is studied numerically through Sameh and Rashed[23]. The existence of magnetic field heat together with mass transfer features on mixed convective copper water nanofluid flow through the inclined plate is investigated in the surrounding porous medium together with viscous dissipation through Uddin *et al.*[24]. Chamkha *et al.*[25] introduced a mathematical model to accentuate the mixed bioconvective flow on a vertical wedge in a Darcy porous medium filled with a nanofluid containing both nanoparticles and gyrotactic microorganisms. Abualnaja *et al.*[26] study the unsteady mixed convection flow along a symmetric wedge embedded in a porous medium saturated with a nanofluid in the presence of first and second-order resistances.

The principal aim of this work is to study the effect of power-law exponent (λ) on the non-dimensional velocity, temperature, nanoparticle volume fraction and Nusselt number has been discussed through graphs. The influence of Brownian motion, buoyancy ratio and thermophoresis parameters on the Nusselt number in the presence of power law exponent is also discussed.

2. Mathematical Formulation

Consider the steady natural convection boundary layer flow past a vertical plate placed in a nanofluid-saturated porous medium. We choose the two-dimensional coordinate system (x,y) such that the x-axis is along with the vertical plate and the y-axis is normal to the plate. The physical flow model and coordinate system are shown in Figure 1. At the surface, the temperature T and the nano-particle fraction φ take constant values T_w and φ_w, respectively. As y tends to infinity, T_∞ and φ_∞ have denoted the ambient values of the temperature and nano-particle fraction, respectively. These ambient values are attained far away from the plate. It is also assumed that the nano-particle concentration is diluted. Assume the homogeneity and local thermal equilibrium in the porous medium whose porosity and permeability are denoted by ε and K, respectively. Then applying the standard boundary layer and Oberbeck-Boussinesq approximations, the governing equations of mass, momentum, thermal energy and nanoparticles fraction can be written as

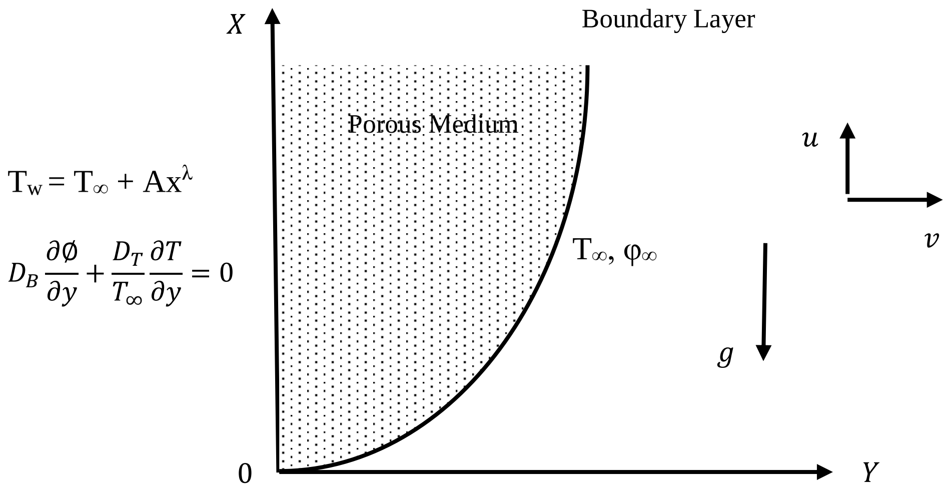

Figure 1: Flow model and physical coordinate system

(1) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2) $$\mathrm{u} = \frac{gK}{\mu}\left[(1-\phi_\infty)(\mathrm{T}-\mathrm{T}_\infty)\beta\rho_{f_\infty} - (\phi-\phi_\infty)\left(\rho_p-\rho_{f_\infty}\right)\right]$$

(3) $$\mathrm{u}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau D_B\left(\frac{\partial T}{\partial y}\frac{\partial \phi}{\partial y}\right) + \frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2$$

(4) $$\frac{1}{\varepsilon}\left(\mathrm{u}\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y}\right) = D_B\frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}$$

where, $\alpha_m = \frac{1}{(\rho c)_f}k_m$ and $\tau = \frac{\varepsilon(\rho c)_p}{(\rho c)_f}$.

In equations (1)-(4), u and v are the velocity components along the x and y axes, g is acceleration due to gravity, μ is effective viscosity of fluid, β is volumetric expansion coefficient of the fluid, ρ_p and ρ_f are nano-particles mass and fluid density respectively. k_m is the effective thermal conductivity of porous medium. α_m is the thermal diffusivity, $(\rho c)_p$ the effective heat capacity of nanoparticles, $(\rho c)_f$ the effective heat capacity of the base fluid, D_B is the Brownian diffusion coefficient and D_T is the thermophoresis diffusion coefficient.

The boundary conditions for the present problem are

(5) $$v = 0, \mathrm{T} = \mathrm{T}_w = T_\infty + Ax^\lambda, D_B\frac{\partial \phi}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0, x \geq 0 \ (\mathrm{y} = 0)$$

(6) $$\mathrm{u} = 0, \mathrm{v} = 0, \mathrm{T} \to \mathrm{T}_\infty, \phi \to \phi_\infty \ (\mathrm{y} \to \infty)$$

where A and λ are positive constants. We assume that the temperature of the vertical plate wall is a power function of the distance from the origin at which it begins to differ from the temperature of the surrounding fluids. Furthermore, we expect the value of the particle fraction on the plate to change according to the absence of nanoparticle flux.
Now we express the following non-dimensional variables by defining

(7) $$\psi = \alpha_m Ra_x^{1/2} s(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \mathrm{f}(\eta) = \frac{\phi-\phi_\infty}{\phi_\infty}, \eta = \left(\frac{y}{x}\right) Ra_x^{1/2}$$

With the local Rayleigh number is defined as

(8) $$Ra_x = \frac{(1-\phi_\infty)\beta g \rho_{f_\infty}}{\mu}\left(\frac{Kx}{\alpha_m}\right)$$

where ψ is a dimensional stream function defined by $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$, and f is a dimensionless stream function, η be the similarity space variable, θ and φ are the dimensionless temperature and nanoparticles volume fraction respectively. Using these dimensionless the continuity equation (1) is satisfied by u and v automatically.

Substituting the dimensionless variables defined in (7) into equations (2)-(4), we have

(9) $$s' + Nrf - \theta = 0$$

(10) $$\theta'' + \left(\frac{\lambda+1}{2}\right) s\theta' + Nbf'\theta' + Nt\theta'^2 = 0$$

(11) $$\mathrm{f}'' + \left(\frac{\lambda+1}{2}\right) Le\, sf' + \left(\frac{Nt}{Nb}\right)\theta'' - \lambda s'\theta = 0$$

Similarly, the boundary conditions (5) and (6) reduce to

(12) $$s(\eta) = 0, \theta(\eta) = 0, \mathrm{Nbf}'(\eta) + \mathrm{Nt}\theta'(\eta) = 0 \text{ at } \eta = 0$$

$$\mathrm{s}'(\eta) \to 0, \theta(\eta) \to 0, \mathrm{f}(\eta) \to 0 \text{ as } \eta \to \infty$$

The prime denotes differentiation with respect to η only. In the above equations we have used the four non-dimensional parameters namely, Brownian motion (Nb), buoyancy ratio (Nr), thermophoresis (Nt) and Lewis number, which are defined as

(13) $$Nb = \frac{\varepsilon D_B(\rho c)_p \phi_\infty}{\alpha_m(\rho c)_f}$$

(14) $$Nr = \frac{(\rho_p - \rho_{f_\infty})\phi_\infty}{\rho_{f_\infty}(1-\phi_\infty)\beta(T_w - T_\infty)}$$

$$Nt = \frac{\varepsilon(\rho c)_p(T_w - T_\infty)D_T}{\alpha_m(\rho c)_f} \tag{15}$$

$$Le = \frac{\alpha_m}{\varepsilon D_B} \tag{16}$$

It is important to note that this boundary value problem reduces to the classical problem solved by Cheng and Minkowycz[6] when λ, *Nb*, *Nr* and *Nt* are zero in equations (9)-(10). The boundary value problem for f then becomes ill-posed and is of no physical significance. The results of practical interest in many applications is the local Nusselt number Nu, which is defined as

$$Nu = \frac{q_w}{(T_w - T_\infty)}\left(\frac{x}{k_m}\right) \tag{17}$$

where q_w is the wall heat flux. The reduced local Nusselt number is given by

$$Nur = Nu/Ra_x^{1/2} = -\theta'(0) \tag{18}$$

3. Results and Discussion

The nonlinear coupled differential equations (9)-(11) along with the boundary conditions (12) from a two point boundary value problem is solved using shooting method. The results compared for the case λ = 0, between those obtained by the study of Kuznetsov and Nield[14,16], Hady *et al.*[17] and the present study for the reduced Nusselt number for different values of Le are given in Table 1. Here C_r, C_b and C_t are the coefficients in the linear regression estimate $Nur = 0.444 + C_rNr + C_bNb + C_tNt$, applicable for Nr, Nb and Nt each in [0, 0.5]. A very good agreement between the results is observed, which confirms that the implemented numerical scheme is accurate.

Table 1: Comparison between the linear regression coefficients for the study[14,16,17] and the present study

	Nield and Kuznetsov[14]			Kuznetsov and Nield[16]		
Le	C_r	C_b	C_t	C_r	C_b	C_t
5	- 0.003	0.004	- 0.090	- 0.003	0.004	- 0.090
10	- 0.001	0.000	- 0.105	- 0.001	0.000	- 0.105
20	- 0.001	- 0.001	- 0.120	- 0.001	- 0.001	- 0.120
50	- 0.002	- 0.002	- 0.135	- 0.002	- 0.002	- 0.135
100	- 0.002	- 0.003	- 0.143	- 0.002	- 0.003	- 0.143
200	- 0.003	- 0.003	- 0.150	- 0.003	- 0.003	- 0.150
500	- 0.003	- 0.003	- 0.155	- 0.003	- 0.003	- 0.155
1000	- 0.003	- 0.003	- 0.158	- 0.003	- 0.003	- 0.158

Hady *et al.*[17]				Present study		
Le	C_r	C_b	C_t	C_r	C_b	C_t
5	- 0.003	0.006	- 0.085	- 0.003	0.006	- 0.085
10	- 0.001	0.000	- 0.104	- 0.001	0.000	- 0.104
20	- 0.001	- 0.001	- 0.118	- 0.001	- 0.001	- 0.118
50	- 0.002	- 0.002	- 0.134	- 0.002	- 0.002	- 0.134
100	- 0.002	- 0.003	- 0.142	- 0.002	- 0.003	- 0.142
200	- 0.003	- 0.003	- 0.149	- 0.003	- 0.003	- 0.149
500	- 0.030	- 0.222	- 0.172	- 0.030	- 0.222	- 0.172
1000	- 0.005	- 0.028	- 0.090	- 0.005	- 0.028	- 0.090

Further, the variation of the linear regression coefficients C_r, C_b and C_t of the reduced Nusselt number Nur for different values of λ = 0.25, 0.50, 0.75 is presented in Table 2.

Table 2: Linear regression coefficients for different values of λ

	λ = 0.25			λ = 0.50			λ = 0.75		
Le	C_r	C_b	C_t	C_r	C_b	C_t	C_r	C_b	C_t
5	0.001	- 0.000	- 0.081	0.005	- 0.004	- 0.077	0.009	- 0.009	- 0.075
10	0.002	-0.004	- 0.099	0.006	- 0.007	-0.099	0.009	- 0.011	- 0.101
20	0.004	- 0.003	- 0.119	0.005	- 0.005	- 0.124	0.007	- 0.008	- 0.129
50	0.007	- 0.008	- 0.137	0.010	- 0.011	- 0.147	0.014	- 0.015	- 0.155
100	0.013	- 0.016	- 0.143	0.026	- 0.035	- 0.104	0.045	- 0.034	- 0.157
200	0.033	- 0.368	- 0.289	0.003	- 0.416	- 0.389	0.009	- 0.464	- 0.371
500	0.065	-0.308	-0.074	0.089	- 0.326	- 0.009	0.225	-0.399	-0.006
1000	0.085	- 0.315	- 0.021	0.067	- 0.398	- 0.019	0.160	- 0.488	- 0.055

For the case *Le* = 20, we calculated 125 sets of values of *Nr*, *Nb* and *Nt* in the range [0.1, 0.2, 0.3, 0.4, 0.5] and the linear regression are performed on the results. These linear regression estimations can be written as

$$(19)\quad \begin{cases} Nur_{est} = 0.444 - 0.004Nr + 0.003Nb - 0.119Nt & (\lambda = 0.25) \\ Nur_{est} = 0.444 - 0.005Nr + 0.005Nb - 0.124Nt & (\lambda = 0.50) \\ Nur_{est} = 0.444 - 0.007Nr + 0.008Nb - 0.129Nt & (\lambda = 0.75) \end{cases}$$

We believe that for most practical purpose the simple linear regression formulas in equation (19) should be enough. Plots the dependent similarity variables $s(\eta), s'(\eta), \theta(\eta)$ and $f(\eta)$ for a typical case of *Le* = 20, *Nr* = 0.5, *Nb* = 0.4, *Nt* = 0.2 and $\lambda = 0$ is shown in figure 2. The shape of the boundary layer profiles for the temperature function $\theta(\eta)$ and stream function $s(\eta)$ are essentially the same as for a regular fluid. In the case of a regular fluid the profile for $s'(\eta)$, which represents the longitudinal component of the velocity u is identical with that for the temperature θ. We now see that in the case of a nanofluid the two profiles diverge when Le is relatively small but tend to coincidence when Le increases. In figures 3-5, we plot the dimensionless temperature, nanoparticle volume fraction and velocity profile for various values of power-law exponent (λ) when Le = 20, Nr = 0.5, Nb = 0.4, Nt = 0.2 and understand the effect of λ.

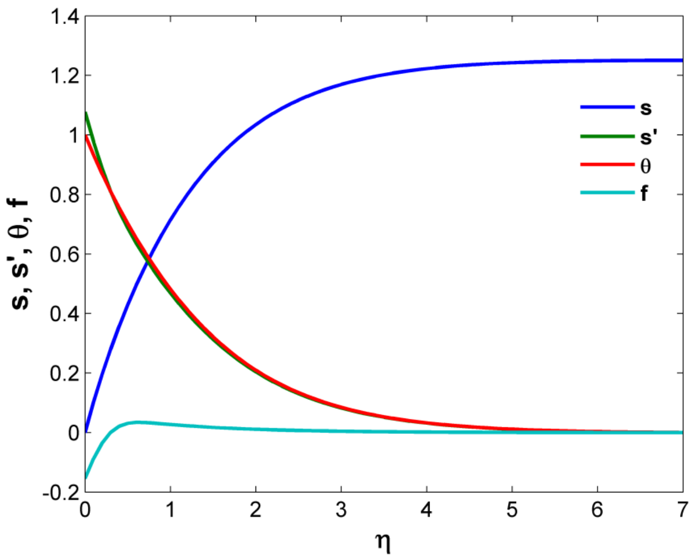

Figure 2: Plots of dimensionless similarity functions $s(\eta), s'(\eta), \theta(\eta)$ and $f(\eta)$ for a typical case of Le = 20, Nr = 0.5, Nb = 0.4, Nt = 0.2 and λ = 0.

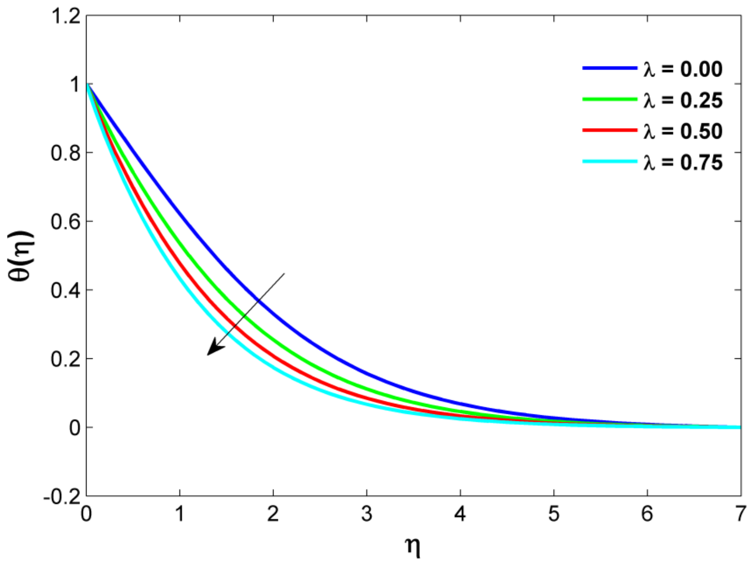

Figure 3: Variation of the temperature function for various values of λ, when Le = 20, Nr = 0.5, Nb = 0.4, Nt = 0.2.

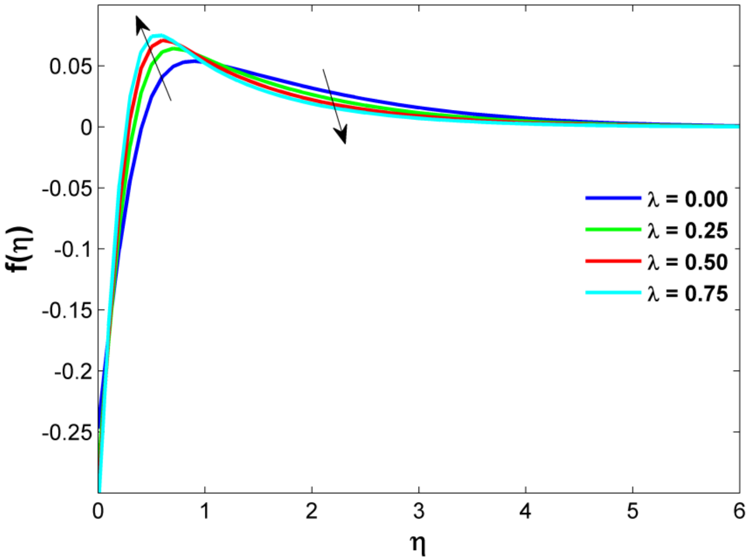

Figure 4: Variation of the nanoparticle volume fraction function for various values of λ, when Le = 20, Nr = 0.5, Nb = 0.4, Nt = 0.2.

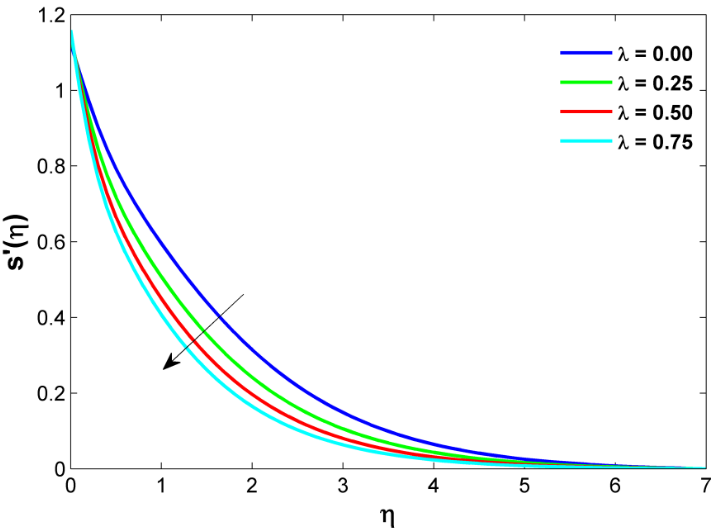

Figure 5: Variation of the velocity function for various values of λ, when Le = 20, Nr = 0.5, Nb = 0.4, Nt = 0.2.

The effect of power-law exponent (λ) on the temperature function is depicted in figure. It shows that the temperature function decreases when power law exponent increases. This happens because power law exponent is directly proportional to the temperature difference. In figure 4, we can see that mass fraction function change their nature alternately within the boundary layer when increasing the power law exponent. The velocity function is shown in figure 5. It is observed that the velocity function is decreases with increasing power law exponent.

6. Conclusion

In this paper, the Cheng-Minkowycz problem was investigated numerically for natural convective boundary layer flow in a porous media soaked with a nanofluid. Brownian motion and thermophoresis parameters are accounted for in the nanofluid model. The Lewis number, the buoyancy ratio, Brownian motion, thermophoresis parameter, and power law exponent are used to generate a similarity solution. The reduced Nusselt number is graphed and tabulated for numerous values of λ and Le. Estimates of the reduced Nusselt number using linear regression are additionally produced when it comes to the buoyancy ratio, Brownian motion, thermophoresis parameter. This study shows that the reduced Nusselt number increases with Brownian motion and decreases with the buoyancy ratio and thermophoresis parameters.

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