AKCE INTERNATIONAL JOURNAL OF GRAPHS AND COMBINATORICS
2025, VOL. 00, NO. 0, 1–5
https://doi.org/10.1080/09728600.2025.2488229

Independent 2-point set domination in graphs with specified girth

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ABSTRACT

A set D of vertices in a connected graph G is said to be an *independent 2-point set dominating set* (or in short i-2psd set) of G if D is an independent set and for every subset $S \subseteq V - D$ there exists a non-empty subset $T \subseteq D$ containing at most 2 vertices such that the induced subgraph $\langle S \cup T \rangle$ is connected. A graph which possesses an i-2psd set is called an i-2psd graph. Every finite graph need not be an i-2psd graph; for instance C_7. In this paper we continue to explore graphs which possess an i-2psd set and discuss i-2psd graphs with specified girth. We exhibit a relation between diameter and girth of an i-2psd graph. We characterize extremal i-2psd graphs with maximum girth, and further present a partial characterization of i-2psd graphs with girth 5.

ARTICLE HISTORY
Received 20 May 2024
Accepted 23 March 2025

KEYWORDS
Independent set; separable graphs; Girth and circumference of graphs; 2-point set domination in graphs; generalised theta graph

AMS SUBJECT CLASSIFICATION:
05C69

1. Introduction

Throughout this paper we consider simple, finite, undirected and connected graphs. For standard terminologies and notation in graph theory used in this paper we refer to the book by West [12] while the terms related to the concept of domination are used in the sense of Haynes et al. [5]

In this paper, G is a simple graph with vertex set $V = V(G)$ and edge set $E = E(G)$. For every vertex $v \in V(G)$, the **open neighborhood** of v is the set $N(v) = \{u \in V(G) \mid uv \in E(G)\}$ and the **closed neighborhood** of v is the set $N[v] = N(v) \cup \{v\}$. The **distance** between two vertices u and v of graph G, denoted by $d(u, v)$, is the length of a shortest path joining them. The **diameter** of G, denoted by $diam(G)$, is given by $\max\{d(u, v) : u, v \in V(G)\}$. The **girth** of a graph G, denoted by $girth(G)$, is the length of a shortest cycle in G. The number of edges in a longest cycle of a graph G is called as the **circumference** of G. A **cycle** of length n is called an n-cycle and is denoted by C_n. A set I of vertices in a graph G is an independent set of G if no two vertices of I are adjacent in G. The **independence number**, [1] $\beta_0(G)$, of a graph G is the maximum cardinality of an independent set.

A **cut-vertex** in a connected graph is a vertex whose removal makes the graph disconnected. A graph having a cut-vertex is called a **separable graph** and a **block** of a graph G is a maximal connected subgraph that has no cut-vertex. A **generalized theta graph**, denoted by $\Theta_{s_1,\ldots,s_k}$, is a graph consisting of a pair of end vertices joined by $k (k \geq 2)$ internally disjoint paths of lengths $s_1, \ldots, s_k \geq 1$.

In [7], the concept of 2-point set dominating set in a graph is defined as follows:

Definition 1.1. [7] A set $D \subseteq V(G)$ is a 2-**point set dominating set** (or, in short, 2-psd set) of G if for every subset $S \subseteq V - D$ there exists a non-empty subset $T \subseteq D$ containing at most 2 vertices such that the induced subgraph $\langle S \cup T \rangle$ is connected. The 2-**point set domination number** $\gamma_{2ps}(G)$ is the minimum cardinality of a 2-psd set of G.

Definition 1.2. [11] A 2-psd set D of a graph G such that the induced subgraph $\langle D \rangle$ is totally disconnected is said to be an **independent 2-psd set** (abbreviated henceforth as i-2psd set) of G.

Unlike independent dominating set, [2,3] every graph does not possess an independent 2psd set. For example the path P_6 does not possess any i-2psd set. Hence characterization of graphs which possess an i-2psd set is an interesting problem. Such a graph is called an i-2psd graph. In [4,11]; the authors have presented several results for i-2psd graphs, which are given below.

Theorem 1.3. [11] If D is an i-2psd set of a graph G, then for any two vertices $x, y \in V - D$, $d(x, y) \leq 2$.

Theorem 1.4. [11] For any graph G possessing an i-2psd set, the following are true:

(a) $diam(G) \leq 4$.
(b) $girth(G) \leq 6$.
(c) $\beta_0(G) \geq \gamma_{2ps}(G)$, where $\beta_0(G)$ is the independence number of the graph G.

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