AKCE INTERNATIONAL JOURNAL OF GRAPHS AND COMBINATORICS
2022, VOL. 19, NO. 3, 201–205
https://doi.org/10.1080/09728600.2022.2094734

Taylor & Francis
Taylor & Francis Group

OPEN ACCESS Check for updates

Independent 2-point set domination in graphs - II

Deepti Jain and Purnima Gupta

Department of Mathematics, Sri Venkateswara College, University of Delhi, Delhi, India

ABSTRACT

A set D of vertices in a connected graph G is said to be an *independent 2-point set dominating set* (or in short i-2psd set) of G if D is an independent set and for every non-empty subset $S \subseteq V - D$ there exists a non-empty subset $T \subseteq D$ having at most 2 vertices such that the induced subgraph $\langle S \cup T \rangle$ is connected. We call a graph to be an i-2psd graph if it possesses an i-2psd set. In this article, we explore i-2psd graphs. We first provide complete structural characterization of separable i-2psd graphs and thereafter, in our quest to characterize i-2psd blocks, we characterize hexagon-free bipartite i-2psd blocks and exhibit a family of non-bipartite i-2psd blocks.

KEYWORDS

Independent set; 2-point set domination; separable graphs

MATHEMATICS SUBJECT CLASSIFICATION

05C69

1. Introduction

Unless defined or mentioned otherwise, we refer the reader to West [8] for standard terminology and notation in graph theory while the terms related to the concept of domination are used in the sense of Haynes et al. [7]. In this article, by a graph we mean a finite, undirected and connected graph without multiple edges or loops.

Let $G = (V, E)$ be a graph. The neighborhood of a vertex v in a graph G, denoted by $N(v)$, is the set of all vertices in G adjacent to v. The set $N(v) \cup \{v\}$ is the closed neighborhood of v in G and is denoted by $N[v]$. The distance between two vertices u and v of G, denoted by $d(u, v)$, is the length of a shortest path joining them. The diameter of G, denoted by $diam(G)$, is given by $\max\{d(u,v) : u, v \in V(G)\}$. The girth of a graph G, denoted by $girth(G)$, is the length of the shortest cycle in G. The independence number, $\beta_0(G)$, of a graph G is the cardinality of the largest independent vertex set. A cycle of length n is called an n-cycle and denoted as C_n. Given a graph H, we say that the graph G is an *H-free graph* [3] if no induced subgraph of G is isomorphic to H. A cut-vertex in a connected graph is a vertex whose removal makes the graph disconnected. A graph having a cut-vertex is called a separable graph and a block of a graph G is a maximal connected subgraph that has no cut-vertex. If G is a separable graph, then the set of all blocks of G is denoted by $\mathcal{B}(G)$ and the set of all non-trivial blocks is denoted by $\mathcal{B}_0(G)$. Further, for a cut-vertex w of a separable graph G, $\mathcal{B}_w(G)$ denotes the set of all blocks at w.

Gupta and Jain in [4, 6] studied 2-point set dominating sets of a graph which is defined as follows:

Definition 1.1. [6] A set $D \subseteq V(G)$ is a 2-point set dominating set (or, in short, 2-psd set) of a graph G if for every non-empty subset $S \subseteq V - D$ there exists a non-empty subset $T \subseteq D$ containing at most 2 vertices such that the induced subgraph $\langle S \cup T \rangle$ is connected. The set of all 2-psd sets of G is denoted by $\mathcal{D}_{2ps}(G)$.

Definition 1.2. [5] A 2-psd set D of a graph G such that the induced subgraph $\langle D \rangle$ is totally disconnected is said to be an **independent 2-psd set** (abbreviated henceforth as i-2psd set) of G. The set of all independent 2-psd sets of G is denoted by $\mathcal{D}_{i2ps}(G)$.

It is noticed in [5] that unlike independent dominating set [1, 2], every graph does not possess an independent 2-psd set. For example, the graph G_1 given in Figure 1 possesses an independent 2-point set dominating set while G_2 does not. Thus the study of graphs which possess an independent 2-psd set is an important problem in the theory of 2-point set domination in graphs.

We call a graph to be an **i-2psd graph** if it possesses an i-2psd set, and otherwise it is referred to as a **non i-2psd graph**. Thus, for an i-2psd graph G, $\mathcal{D}_{i2ps}(G) \neq \phi$.

In [5], Gupta and Jain considered the problem of characterizing connected graphs having an i-2psd set. They provided with some necessary and some sufficient conditions for a graph to be an i-2psd graph and further characterized some classes of i-2psd graphs. The following are some results in this direction mentioned in [5].

Theorem 1.3. [5] *For any graph G, an independent subset $D \subseteq V$ is an i-2psd set of G if and only if for every independent subset $S \subseteq V - D$ there exists a subset $T \subseteq D$ such that $|T| \leq 2$ and $\langle S \cup T \rangle$ is connected.*

Theorem 1.4. [5] *If D is an i-2psd set of a graph G, then for any two vertices $x, y \in V - D$, $d(x, y) \leq 2$.*

Theorem 1.5. [5] *For any graph G possessing an i-2psd set, the following are true:*

CONTACT Deepti Jain djain@svc.ac.in Department of Mathematics, Sri Venkateswara College, University of Delhi, Delhi, India.