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Graphoidally independent infinite graphs

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ABSTRACT

A graphoidal cover of a graph G (not necessarily finite) is a collection ψ of paths in G, called ψ-edges, (not necessarily finite, not necessarily open) satisfying the following axioms: (GC-1) Every vertex of G is an internal vertex of at most one path in ψ, and (GC-2) every edge of G is in exactly one path in ψ. The pair (G,ψ) is called a graphoidally covered graph. In a graphoidally covered graph (G,ψ), two distinct vertices u and v of G are ψ-adjacent if they are the ends of a finite open path in ψ. A graphoidally covered graph (G,ψ) (or G) in which no two distinct vertices are ψ-adjacent is called ψ-independent and a graph G possessing a graphoidal cover ψ such that G is ψ-independent is called a graphoidally independent graph. This paper is an attempt to characterize graphoidally independent infinite graphs. In this paper we establish complete characterization of graphoidally independent infinite trees, infinite unicyclic graphs and infinite 2-edge connected graphs.

KEYWORDS

Graphoidal cover of a graph; graphoidally covered graphs; graphoidally independent graphs

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1. Introduction

Throughout this paper, unless specifically mentioned otherwise, notation and terminology in graph theory will be as in West [14], except that a graph could be infinite in which case the reader is referred to Ore [11].

The concept of graphoidal covers for finite graphs was first introduced by Acharya and Sampathkumar [1] in 1987 as a close variant of another emerging discrete structure called semigraphs [13]. Many interesting notions like graphoidal covering number [1], graphoidal labeling [10], graphoidal length [9], etc., based on graphoidal covers of a graph were introduced and are being studied extensively. In particular, the notion of graphoidal covering number of a graph has attracted many researchers and numerous work is present in the literature [4–7, 12]. A detailed treatment of graphoidal covers and graphoidally covered graphs is given in [8]. Later on in 1999, Acharya and Gupta [2] extended the notion of graphoidal covers to infinite graphs.

Given a graph $G=(V,E)$ (not necessarily finite) by a *graphoidal cover of* G we mean a collection ψ of non-trivial paths in G, which are not necessarily open and not necessarily finite, satisfying the following axioms: (GC-1) Every vertex of G is an internal vertex of at most one path in ψ, and (GC-2) Every edge of G belongs to exactly one path in ψ.

For a given graphoidal cover ψ of a graph G, the paths in ψ are called *ψ-edges* of G, and the ordered pair (G,ψ) is called a *graphoidally covered graph*. In this definition, when G is infinite, a ψ-edge could possibly be infinite; in particular, it may be a one-way infinite path having one end-vertex or a two-way infinite path having no end-vertex. Further, a finite open ψ-edge has two distinct end-vertices. A closed ψ-edge is a cycle and has only one coincident end-vertex, which is specified by ψ.

The set of all graphoidal covers of G is denoted by $\mathcal{G}_G$. Clearly, $E =: E(G)$, the edge set of G, is a graphoidal cover of G and is called the trivial graphoidal cover of G; a graphoidal cover that is not trivial will be referred to as being non-trivial.

An interesting theoretical motivation for studying graphoidally covered graphs is that every ear-decomposition of a finite 2-connected (2-edge connected) graph G (cf.: [14], p.146) is a graphoidal cover of G, thus, the notion of a graphoidal cover of any graph can also be looked upon as a generalization of the notion of an ear-decomposition of a finite 2-connected(2-edge connected) graph. Graphoidal cover of a graph can also be viewed as a collection of internally vertex disjoint paths covering the edge set of the graph and two vertices are adjacent when they are the endpoints of a path of the collection; hence, one may view the notion of graphoidal covers of a graph as a graph decomposition into internally vertex-disjoint paths of a graph, which not only holds for finite graphs but also for infinite graphs.

Definition 1.1. In a graphoidally covered graph (G,ψ), two distinct vertices u and v of G are said to be **ψ-adjacent** if they are the ends of an open ψ-edge; a vertex is said to be self-ψ-adjacent if it is both the 'initial' and the 'terminal' end of a closed ψ-edge (i.e., a cycle in ψ).

A graphoidal cover ψ of a graph G is called a **totally disconnecting graphoidal cover** of G if no two distinct vertices in G are ψ-adjacent. Clearly, for a graphoidally covered

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