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Bounds on 2-point set domination number of a graph

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A set $D \subseteq V(G)$ is a 2-*point set dominating set* (2-psd set) of a graph G if for any subset $S \subseteq V - D$, there exists a nonempty subset $T \subseteq D$ containing at most two vertices such that the subgraph $\langle S \cup T \rangle$ induced by $S \cup T$ is connected. The 2-*point set domination number* of G, denoted by $\gamma_{2\text{ps}}(G)$, is the minimum cardinality of a 2-psd set of G. In this paper, we determine the lower bounds and an upper bound on $\gamma_{2\text{ps}}(G)$ of a graph. We also characterize extremal graphs for the lower bounds and identify some well-known classes of both separable and nonseparable graphs attaining the upper bound.

Keywords: Domination; point-set domination; 2-point set domination; split graph; block-cactus graph; middle graph.

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1. Introduction

In this paper, we consider finite, undirected and connected graphs without multiple edges and loops. The sets $V(G)$ and $E(G)$ denote the vertex set and the edge set of a graph G, respectively. The order $|V(G)|$ of G is denoted by n. The open neighborhood $N(v)$ of a vertex v in G is the set of vertices adjacent to v in G and the closed neighborhood $N[v]$ of a vertex v is the set $N(v) \cup \{v\}$. The degree $|N(v)|$ of a vertex $v \in V(G)$ in the graph G is denoted by $\deg(v)$ and the maximum degree in the graph G is denoted by $\Delta(G)$. For a set $S \subseteq V(G)$ the open (closed) neighborhood $N(S)$ $(N[S])$ of S in G is defined as $N(S) = \cup_{v \in S} N(v)$ $(N[S] = \cup_{v \in S} N[v])$.

In a connected graph G, the *distance* between two vertices u and v is the length of the shortest path joining u and v, and is denoted by $d(u, v)$. A subgraph H of G is said to be an *induced subgraph of* G if each edge of G having its ends in $V(H)$ is