

Independent 2-Point Set Domination in Graphs

Purnima Gupta and Deepti Jain^(✉)

Department of Mathematics, Sri Venkateswara College, University of Delhi,
New Delhi 110021, Delhi, India
purnimachandni1@gmail.com, djain@svc.ac.in

Abstract. A set D of vertices in a connected graph G is said to be an *independent 2-point set dominating set* (or in short i-2psd set) of G if D is an independent set and for every subset $S \subseteq V - D$ there exists a non-empty subset $T \subseteq D$ containing at most 2 vertices such that the induced subgraph $\langle S \cup T \rangle$ is connected. In this paper we explore graphs which possess an i-2psd set.

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1 Introduction

By a graph G we mean a finite, undirected, connected and non-trivial graph with neither loops nor multiple edges. The order $|V(G)|$ and the size $|E(G)|$ of G are denoted by n and m respectively. For graph theoretic terminology we refer to West [9]. For the terms related to the concept of domination we refer to Haynes et al. [6].

In [8], Gupta and Jain define 2-point set domination in a graph as follows:

Definition 1. A set $D \subseteq V(G)$ is a 2-point set dominating set (or, in short, 2-psd set) of G if for every subset $S \subseteq V - D$ there exists a non-empty subset $T \subseteq D$ containing at most 2 vertices such that $\langle S \cup T \rangle$ is connected. The 2-point set domination number of G , denoted by $\gamma_{2ps}(G)$, is the minimum cardinality of a 2-psd set of G .

Definition 2. A 2-psd set D of a graph G which is also independent is said to be an **independent 2-psd set** (abbreviated henceforth as i-2psd set) of G .

The theory of independent domination was formalized by Berge [3] and Ore [7]. The independent domination number and the notation $i(G)$ were introduced by Cockayne and Hedetniemi in [4, 5]. It is well known that *every finite graph has an independent dominating set*. However, such a statement is not true in the case of 2-psd sets in graphs. For example, the cycle C_7 has no independent 2-psd set.

We call a graph to be an **i-2psd graph** if it possesses an independent 2-psd set and a **non i-2psd graph** otherwise. Obviously, if G is a graph with a vertex of full degree, then G possesses an i-2psd set. Further, every complete bipartite graph $K_{m,n}$ is an i-2psd graph. The main aim of this paper is to explore graphs which possess an independent 2-psd set.