

2-POINT SET DOMINATION NUMBER OF A CACTUS

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ABSTRACT. A set $D \subseteq V(G)$ is a *2-point set dominating set* (2-psd set) of a graph G if for any subset $S \subseteq V - D$, there exists a non-empty subset $T \subseteq D$ containing at most two vertices such that the subgraph $\langle S \cup T \rangle$ induced by $S \cup T$ is connected. The *2-point set domination number* of G , denoted by $\gamma_{2ps}(G)$, is the minimum cardinality of a 2-psd set of G . In this paper we give the 2-psd number $\gamma_{2ps}(G)$ for cactus. Also we give an alternate proof for $\gamma_{2ps}(G)$ of unicyclic graphs which also reflects the structure of minimum 2-psd sets of unicyclic graphs.

1. INTRODUCTION AND PRELIMINARIES

Unless defined or mentioned otherwise, we refer the reader to West [7] for standard terminology and notation in the theory of graphs. Also, in this paper by a graph we mean a finite, simple, undirected and connected graph.

The vertex set of a graph G is denoted by $V(G)$ and $n = |V(G)|$. For any vertex $v \in V(G)$, the *open neighborhood* $N(v)$ of v in G is the set $\{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* $N[v]$ of v in G is the set $N(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the *open neighborhood* $N(S)$ of S in G is $\cup_{v \in S} N(v)$ and the *closed neighborhood* $N[S]$ of S in G is $N(S) \cup S$.

In a connected graph G , the *distance* between two vertices u and v is the length of the shortest path joining u and v , and is denoted by $d(u, v)$. A subgraph H of a graph G is said to be *induced* if, for any pair of vertices x and y of H , xy is an edge of H if and only if xy is an edge of G . If the vertex set of H is the subset S of $V(G)$, then H can be written as $\langle S \rangle$ and is said to be *induced by S* .

Definition 1.1. A set $D \subseteq V(G)$ in a graph G is said to be a *dominating set* of G if for every vertex v in $V - D$, there exists a vertex $u \in D$ such that v is adjacent to u . The *domination number* of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G (See [3, 4]).

Various types of dominations, such as set domination [6] and point-set domination [1, 2, 5] have been widely studied.

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