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On graphs whose graphoidal domination number is one

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Abstract

Given a graph $G = (V, E)$, a set ψ of non-trivial paths, which are not necessarily open, called ψ-edges, is called a *graphoidal cover of* G if it satisfies the following conditions: $(GC-1)$ Every vertex of G is an internal vertex of at most one path in ψ, and $(GC-2)$ every edge of G is in exactly one path in ψ; the ordered pair (G, ψ) is called a *graphoidally covered graph*. Two vertices u and v of G are *ψ-adjacent* if they are the ends of an open ψ-edge. A set D of vertices in (G, ψ) is *ψ-dominating (in short ψ-dom set)* if every vertex of G is either in D or is ψ-adjacent to a vertex in D. Let $\gamma_\psi(G) = \inf\{|D| : Dis a \psi - dom set of G\}$. A ψ-dom set D with $|D| = \gamma_\psi(G)$ is called a $\gamma_\psi(G)$-set. The *graphoidal domination number of a graph* G denoted by $\gamma_\psi^0(G)$ is defined as $inf\{\gamma_\psi(G) : \psi \in \mathcal{G}_G\}$. Let G be a connected graph with cyclomatic number $\mu(G) = (q - p + 1)$. In this paper, we characterize graphs for which there exists a non-trivial graphoidal cover ψ such that $\gamma_\psi(G) = 1$ and $l(P) > 1$ for each $P \in \psi$ and in this process we prove that the only such graphoidal covers are such that $l(P) = 2$ for each $P \in \psi$.

Keywords: Graphoidal cover; Graphoidally covered graph; ψ-domination

1. Introduction

Throughout this paper, we shall follow the notation and terminology of Harary [1] and West [2] for graphs, unless specifically defined otherwise. Also, all graphs considered in this paper are simple, finite, undirected and connected. Let $G = (V, E)$ be a graph. The number of vertices and number of edges of G are denoted by p and q respectively. If $P = (u_0, u_1, u_2, \ldots, u_n)$ is a path, not necessarily open, in G then u_0 and u_n are called *external or end-vertices* of P and $u_1, u_2, \ldots, u_{n-1}$ are called *internal vertices* of P. Given a graph $G = (V, E)$, a set ψ of non-trivial paths, which are not necessarily open, called *ψ-edges*, is called a *graphoidal cover of* G if it satisfies the following conditions: $(GC-1)$ Every vertex of G is an internal vertex of at most one path in ψ, and $(GC-2)$ every edge of G is in exactly one path in ψ; the ordered pair (G, ψ) is called a *graphoidally covered graph*. The set of all graphoidal covers of a graph G is denoted by $\mathcal{G}_G$. For the study of graphoidal covers, the reader can refer to [3,4].

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