International Journal of Modelling and Simulation

ISSN: 0228-6203 (Print) 1925-7082 (Online) Journal homepage: www.tandfonline.com/journals/tjms20

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To cite this article: Pranay Goswami, Arushi & Bhawna Pokhriyal (09 Jul 2026): Analytical solution of generalized Bratu type equations with the Le Roy-type Mittag-Leffler function, International Journal of Modelling and Simulation, DOI: 10.1080/02286203.2026.2699153

To link to this article: https://doi.org/10.1080/02286203.2026.2699153

Published online: 09 Jul 2026.

INTERNATIONAL JOURNAL OF MODELLING AND SIMULATION
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Analytical solution of generalized Bratu type equations with the Le Roy-type Mittag-Leffler function

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ABSTRACT

This work generalized the differential equation by incorporating the three-parameter Le Roy-type Mittag-Leffler function and generalized fractional derivative operators into the traditional Bratu equation. Furthermore, using the homotopy perturbation transform method to solve nonlinear fractional differential equations. In all cases, the HPTM series solutions demonstrated rapid convergence, with the second- or third-order approximations often nearly coinciding, numerically validating the convergence analysis. The existence, uniqueness, and convergence of the solution are thoroughly established. Particular extraordinary circumstances are thoroughly examined. The proposed method offers a comprehensive approach to solving generalised Bratu-type equations and provides new insights into the behaviour of fractional systems exhibiting intricate nonlinear dynamics.

ARTICLE HISTORY
Received 9 November 2025
Accepted 18 June 2026

KEYWORDS
Fractional Bratu-type equation; Le Roy function; generalized FDE; HPTM; Mittag-Leffler function

MSC
Primary 34A34; 65M06; Secondary 26A33; 33E12

1. Introduction

In both pure and applied mathematics, fractional calculus (FC), which examines derivatives and integrals of non-integer order, is a fast-growing area of research. It has become a key mathematical tool for describing systems with memory and behaviors that are not local in the last few decades. Biology, physics, chemical engineering, and materials science are just a few examples of scientific and engineering fields that typically contain similar qualities. The increasing prevalence of fractional differential equations (FDEs) can be attributed to their superior capacity to model the dynamics of complex real-world systems, compared to conventional integer-order models [1–4]. The extension of classical differential operators to fractional orders has enabled the development of more versatile and accurate mathematical models, particularly for systems influenced by processes such as diffusion, viscoelasticity, anomalous transport, and others that are insufficiently represented by traditional calculus. The Bratu-type equation has garnered significant interest due to its applicability in combustion theory, thermal analysis, chemical kinetics, and nonlinear engineering challenges, among several applications of fractional differential equations [5,6].

Many equations do not have easy-to-write solutions; therefore, new analytical and numerical methods need to be developed. As a result, a number of semi-analytical and approximate methods have been developed, including the Adomian Decomposition Method (ADM), the Homotopy Analysis Method (HAM), the Generalized Differential Transform Method (GDTM), the Homotopy Perturbation Method (HPM), the Modified Laplace Decomposition Method (MLDM), the Homotopy Perturbation Transform Method (HPTM), and the q-Homotopy Analysis Transform Method (q-HATM). Each of these methods offers an edge for handling non-linearities in fractional systems. Among these, the Homotopy Perturbation Method (HPM), introduced by J.-H. He [7–9], is noticeable due to its plainness, strong convergence guarantees, and capability of creating analytical series solutions that nearly approximate exact solutions. The HPM and its variants, such as HPTM, have been effectively applied to a wide range of nonlinear fractional problems, demonstrating their effectiveness in both theoretical and applied contexts.

The classical Bratu problem, which emerged from thermal combustion theory, has been studied thoroughly through various analytical and numerical methods [10–12]. More recent research has extended the Bratu equation to fractional-order forms, frequently using methods like variational iteration, Adomian decomposition, and homotopy-inspired methods to get exact or approximate analytical solutions [13–16]. However, in most of these studies, the scope of

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