



A study of Bogoyavlenskii's (2+1)-dimensional breaking soliton equation: Lie symmetry, dynamical behaviors and closed-form solutions

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ABSTRACT

This paper employs the Lie symmetry analysis to investigate novel closed-form solutions to a (2+1)-dimensional Bogoyavlenskii's breaking soliton equation. This Lie symmetry technique, used in combination with Maple's symbolic computation system, demonstrates that the Lie infinitesimals are dependent on five arbitrary parameters and two independent arbitrary functions $f_1(t)$ and $f_2(t)$. The invariance criteria of Lie group analysis are used to construct all infinitesimal vectors, commutative relations of their examined vectors, a one-dimensional optimal system and then several symmetry reductions. Subsequently, Bogoyavlenskii's breaking soliton (BBS) equation is reduced into several nonlinear ODEs by employing desirable Lie symmetry reductions through optimal system. Explicit exact solutions in terms of arbitrary independent functions and other constants are obtained as a result of solving the nonlinear ODEs. These established results are entirely new and dissimilar from the previous findings in the literature. The physical behaviors of the gained solutions illustrate the dynamical wave structures of multiple solitons, curved-shaped wave-wave interaction profiles, oscillating periodic solitary waves, doubly-solitons, kink-type waves, W-shaped solitons, and novel solitary waves solutions through 3D plots by selecting the suitable values for arbitrary functional parameters and free parameters based on numerical simulation. Eventually, the derived results verify the efficiency, trustworthiness, and credibility of the considered method.

Introduction

Nonlinear partial differential equations (NPDEs) are widely used to interpret many complicated areas of mathematical physics and engineering sciences, such as optical fibers, fluid dynamics, mathematical biology, plasma physics, geochemistry, oceanography, chemical physics, condensed matter physics, and others. The exact solutions provided the specific information of the physical systems needed to investigate the dynamical behavior of such equations. Thus, the exact invariant-form solutions of NPDEs play a significant role in explaining the various nonlinear complex physical models. Therefore, several mathematical methods like extended tanh method [1], Inverse scattering method [2,3], Exp-function method [4], generalized exponential rational function method [5–7], Bäcklund transformation method [8,9], Lax pair method [10], Factor method [11], Jacobi elliptic method [12],

Hirota method [13–15], Generalized conditional symmetry [16], Hyperbolic tangent method [17], F-expansion method [18], Homogeneous balance method [19], simplest equation technique [20], Riccati–Bernoulli technique [21,22], Lie symmetry analysis technique [23–30], and many other have been developed to handling the nonlinear PDEs.

In recent decades, the Lie group of transformation method has emerged as the most powerful, efficient, and trusted mathematical tool for determining exact analytic solutions to nonlinear PDEs. Lie symmetry reductions allow us to reduce the number of independent variables in higher-dimensional nonlinear PDEs using this technique. Furthermore, for a one-parameter Lie group of transformations, this technique is based on the invariance property. As a result, the Lie symmetry approach is a widely used analytical mathematical tool for obtaining explicit closed-form solutions to nonlinear PDEs, with numerous

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