



Some more closed-form invariant solutions and dynamical behavior of multiple solitons for the (2+1)-dimensional rdDym equation using the Lie symmetry approach

Sachin Kumar^{a,*}, Hassan Almusawa^b, Amit Kumar^c

^a Department of Mathematics, Faculty of Mathematical Sciences, University of Delhi, Delhi 110007, India

^b Department of Mathematics, College of Sciences, Jazan University, Jazan 45142, Saudi Arabia

^c Department of Mathematics, Sri Venkateswara College, University of Delhi, Delhi 110021, India

ARTICLE INFO

Keywords:

(2+1)-dimensional rdDym equation
Lie symmetry technique
Symmetry reductions
Exact closed-form solutions
Solitons

ABSTRACT

The present study is devoted to obtaining some exact generalized solutions and new solitary wave solutions for a (2+1)-dimensional nonlinear r th dispersionless Dym (rdDym) equation by utilizing the Lie symmetry analysis. The Lie infinitesimals, geometric vector fields, the commutation relation of the symmetry Lie algebra are derived by employing the Lie group symmetry technique. Thenceforth, explicit exact closed-form solutions are achieved through two stages of symmetry reductions. These closed-form solutions include arbitrary independent functional parameters and other constant parameters, therefore exhibiting rich localized solitons and comprising the existing solutions in the literature. Furthermore, the different dynamical wave structures of established soliton solutions in the forms of oscillating periodic multisolitons, the interaction between four lump-solitons with parabolic waves, W-shaped solitons, and the interaction between two-doubly solitons with parabolic waves, periodic multi-solitons, and parabolic-shaped solitons profiles are exhibited graphically through three-dimensional plots.

Introduction

Many complex physical phenomena and dynamical structures of solitons in condensed matter physics, plasma physics, ion-acoustics, optical physics, oceanography, mathematical chemistry, and mathematical biology can be represented by nonlinear evolution equations (NLEEs). Whenever we want to understand the dynamical behaviors of complex physical phenomena, described by NLEEs, closed-form invariant solutions for the NLEEs have to be examined. Accordingly, it is more vital to find the invariant closed-form soliton solutions of the corresponding NLEEs explaining the evolution of such nonlinear models. The exact closed-form solutions of the NLEEs provide plenty of information about such equations through dynamical structures and characteristics of resulting soliton solutions.

Nowadays, dynamics of solitary wave solutions are studied in various areas of mathematical sciences. Many mathematicians and new researchers focus on finding the different types of soliton solutions and the dynamics of the soliton solutions for NLEEs with the help of the ansatz solitary wave method [1,2]. Various forms of localized solitary wave solutions are demonstrated for NLEEs. Among these forms: the periodic solitary wave solutions, oscillating multi-solitons, Lump

waves, cnoidal waves, single solitons, doubly solitons, and interaction of soliton solutions. A variety of analytical methods for finding the exact analytic solutions of NLEEs and for illustrating the dynamics of multiple solitary wave solutions have been developed. One of the best techniques for seeking exact analytical solutions of the NLEEs is the Lie symmetry technique [3,4]. The fundamental standards on the ideology of Lie point symmetries that are associated with Lie groups of continuous-point transformations. Sophus Lie, at the end of the 19th century, discovered the Lie group of transformation method, and his pioneer findings well described the theory of transformations groups. Lie group theoretic method is the most powerful, extensively, and efficiently implemented technique for recent research works [5–11] in mathematical sciences and other fields of engineering sciences.

The present paper deals with the investigation of a (2 + 1)-dimensional rdDym equation via the Lie-group symmetry approach. The (2+1)-dimensional r th dispersionless Dym (rdDym) equation is

$$u_{ty} - u_x u_{xy} + u_y u_{xx} = 0, \quad (1)$$

which is introduced by Morozov [12] and investigated as reductions of generalization which include the general r th dispersionless Dym

* Corresponding author.

E-mail address: sachinambariya@gmail.com (S. Kumar).

<https://doi.org/10.1016/j.rinp.2021.104201>

Received 15 March 2021; Received in revised form 8 April 2021; Accepted 11 April 2021

Available online 17 April 2021

2211-3797/© 2021 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).