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Lie symmetry analysis for obtaining the abundant exact solutions, optimal system and dynamics of solitons for a higher-dimensional Fokas equation

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ABSTRACT

In this article, the Lie group of transformation method via one-dimensional optimal system is proposed to obtain some more exact solutions of the (4+1)-dimensional Fokas equation. Lie infinitesimal generators, possible vector fields, and their commutative and adjoint relations are presented by employing the Lie symmetry method. An optimal system of the one-dimensional subalgebras is also constructed using Lie vectors. Meanwhile, based on the optimal system, Lie symmetry reductions of the Fokas equation is obtained. A repeated process of Lie symmetry reductions, using the single, double, triple, quadruple, and quintuple combinations between the considered vectors, transforms the Fokas equation into nonlinear ordinary differential equations which produce abundant group-invariant solutions. The same problem was studied by Sadat et al. (Chaos Solitons Fractals 140:110134, 2020) using the same Lie symmetry technique via commutative product approach but with the less number of vector fields and therefore could obtain only three exact solutions as compared to the number of analytic solutions in this paper.

In order to provide rich localized structures, some solutions are supplemented via numerical simulation, which produces some breather-type solitons, oscillating multi-solitons on the parabolic-shaped surface, fractal dromions, lump-type solitons, and annihilation of different parabolic multi-solitons profiles. The dynamical behaviors of excitation-localized structures are demonstrated graphically via 3D plots for suitable values of the arbitrary free parameters and independent arbitrary functions.

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1. Introduction

The investigation of the exact analytic solutions of the nonlinear partial differential equations (PDEs), which play an important role to understand the various qualitative and quantitative features of nonlinear phenomena in various fields of mathematical sciences such as plasma physics, optical fibers, high energy physics, chemical reactions, shallow water waves, nonlinear dynamics, soliton theory, etc. This is why, numerous computational methods like Lie group method [1,2], Bäcklund transformation method [3], Jacobi elliptic method [4], Inverse scattering method [5], Hirota method [6,7], Exp-function method [8], Generalized exponential rational function method [9], F-expansion method [10], Hyperbolic tangent method [11], simplest equation method [12], Riccati-Bernoulli method [13,14], Homogeneous balance method

[15], (G'/G)-expansion method [16–18], and many more have been developed for obtaining the exact closed-form solutions of the nonlinear PDEs [19–22,39,44].

Lie symmetry method is one of the efficient methods to obtain exact analytic solutions of nonlinear PDEs and was introduced by Norwegian mathematician Marcus Sophus Lie. This Lie symmetry method becomes one of the powerful techniques to get the invariant solutions and exact soliton solutions of nonlinear PDEs. Over the last decades, the Lie symmetry method has been implemented to many models in mathematical sciences and engineering physics (see, e.g., [23,24]). Usually, research scientists and mathematicians focus on lower-dimensional physical models, particularly nonlinear evolution equations in (1+1)-dimensions, (2+1)-dimensions, and (3+1)-dimensions. As we know that there are very rarely a few studies on higher-dimensional equations due to their complexity. Therefore, we would like to explore one higher-dimensional Fokas equation in (4+1)-dimensions and make connections with reduced

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