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Lie symmetries, optimal system and group-invariant solutions of the (3+1)-dimensional generalized KP equation

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ABSTRACT

By applying the Lie symmetry method, abundant group-invariant solutions are constructed for a (3+1)-dimensional generalized Kadomtsev-Petviashvili (gKP) equation, which provides a water wave model for long waves of small amplitude with weakly non-linear restoring forces and frequency dispersion. Infinitesimal generators of symmetries, the commutation table of the symmetry Lie algebra, and an optimal system of one-dimensional Lie symmetry sub-algebras are presented. The governing gKP equation is reduced into several nonlinear ordinary differential equations utilizing thrice symmetry reductions. Consequently, abundant group invariant solutions are obtained in the shapes of different dynamical wave structures of solitons, multi-solitons, W-shaped solitons, doubly solitons, kink-type solitons, lump-type solitons interaction between parabolic waves and lump solitons, and annihilation multi-solitons profiles. The physical interpretation of the resulting soliton solutions is illustrated by three dimensional graphical through numerical simulation. The obtained group-invariant solutions involve many arbitrary functions, thereby exhibiting rich physical structures and including the existing solutions in the literature.

1. Introduction

Nonlinear partial differential equations (PDEs) appearing in various physical fields like plasma physics, oceanography, condensed matter physics, fiber optics, fluid dynamics and marine engineering demonstrate a rich range of nonlinear complex phenomena. It is a well-known fact that it is one of the hottest topics in mathematical and physical sciences to seek exact analytic solutions of nonlinear PDEs. In the study on nonlinear PDEs, it has increasing importance to understand the dynamical behavior of solutions through qualitative and quantitative characteristics of nonlinear equations. To further understand nonlinear complex phenomena, with the help of symbolic computation, numerous powerful methods are introduced to investigation of nonlinear PDEs, including the inverse scattering transform [1], Bäcklund and Darboux transformations [2], the Hirota bilinear method [3], homotopy analysis method [4,5], the generalized bilinear method [6,7] and the Lie symmetry method [8].

Lie symmetry analysis is among the most systematic methods to obtain exact closed-form solutions to nonlinear PDEs of

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