

# Group invariant solutions of (2+1)-dimensional rdDym equation using optimal system of Lie subalgebra

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## Abstract

We use the Lie group method to investigate a one-dimensional optimal system of Lie subalgebra for a (2+1)-dimensional rdDym equation. The new closed form solutions of the rdDym equation are constructed. The rdDym equation has been changed into a new partial differential equation which has fewer independent variables. To explain the physical significance of invariant solutions, 3D, 2D and contour plots are exploited by numerical simulation. Some of the results are new as compared with previously established outcomes (Baran *et al* 2016 *Theor. Math. Phys.* **188** 1273–1295; Morozov 2012 *SIGMA* **8** 051; Lelito and Morozov 2018 *J. Geom. Phys.* **131** 89–100). Interestingly, one lump-type solution, kink waves and a parabolic profile of the group invariant solutions are shown to make this research more valuable.

Keywords: (2+1)-dimensional rdDym equation, Lie symmetry analysis, optimal system, invariant solutions

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Our universe is governed by nonlinearities in complex phenomena of nature, with a wide range of physical activities, and mathematical applications. This research work presents the cause of complexity in nature by general integrable nonlinear evolution equations. A class of nonlinear partial differential equations (PDEs) appear in many areas such as mathematical physics, fluid mechanics, plasma physics, quantum electrodynamics, nuclear and atomic physics, mathematical biology, and so on [1–5]. There are many analytical and numerical methods which have been proposed for finding the exact solutions of nonlinear PDEs for better understanding the

nonlinearities. A few powerful methods are used for obtaining exact solutions in closed form, such as the Darboux transformation method [6], homogeneous balance method [7], Hirota's bilinear method [8], the truncated Painlevé expansion [9], inverse scattering theory [10], tanh function method [11], and so on. One of the famous methods proposed by Sophus Lie is 'one parameter Lie group of transformation method'.

One of the most significant extensions relates to Lie's work on obtaining invariant solutions for PDEs, beginning with the method of partially invariant solutions of Ovsiannikov [12]. A point symmetry of a PDE system maps each of its solutions into a one-parameter family of solutions. This method identifies variable transformations by which new solutions can be constructed from existing ones by using a differential operator, that is, the generator of the symmetry

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